

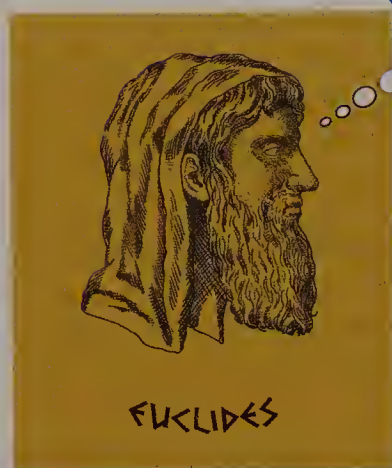
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A Beginning of Geometry

- 1.1** Tips for Success in Mathematics
- 1.2** Geometry—A Mathematical System
- 1.3** Points, Lines, and Planes
- 1.4** Segments and Their Measure
- 1.5** Angles and Their Measure
- 1.6** Angle Pairs and Their Relationships
- 1.7** Coordinate Geometry—Midpoint and Distance Formulas
- 1.8** Constructions—Basic Geometry Constructions

Below is an attempt to show a beginning of Euclidean Geometry. I hope you see that we are only scratching the surface in this text.

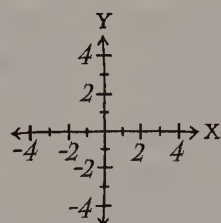
Geometry arose independently in many early cultures as a practical need for consistent surveying and measuring. The Greek philosopher Thales (6th century BC) is given credit for starting to formalize this topic.



I'll call it the Parallel Postulate.

Euclid was a Greek mathematician (300 BC) and known as the "Father of Geometry." He is given credit for organizing known geometry (and number theory) in a written work called *Elements*. No other geometry existed for over 2000 years!

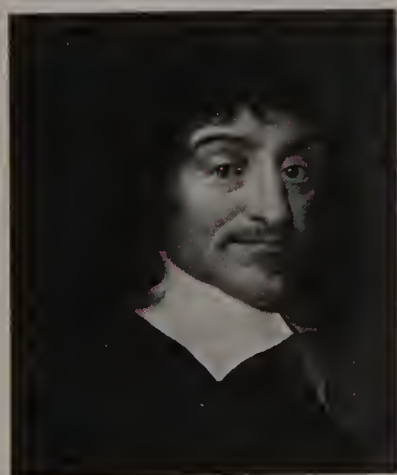
I wonder if this will work?



What is the geometry of our universe? See the Historical Note at the end of this chapter.



During the reign of Euclid's geometry, the 17th century mathematician René Descartes merged geometry and algebra. He "invented" the coordinate system we still use, called the plane coordinate system or the Cartesian coordinate system.



1.1 TIPS FOR SUCCESS IN MATHEMATICS

OBJECTIVES

- 1 Get Ready for This Course.
- 2 Understand Some General Tips for Success.
- 3 Understand How to Use the Resources Provided in MyMathLab and Math XL.
- 4 Get Help as Soon as You Need It.
- 5 Learn How to Prepare for and Take an Exam.
- 6 Develop Good Time Management.

Before reading this section, remember that your instructor (or teacher) is your best source for information. Please see your instructor (or teacher) for any additional help or information.

OBJECTIVE 1 ▶ Getting Ready for This Course. Now that you have decided to take this course, remember that a *positive attitude* will make all the difference in the world. Your belief that you can succeed is just as important as your commitment to this course. Make sure you are ready for this course by having the time and positive attitude that it takes to succeed.

Next, make sure you have scheduled your math course at a time that will give you the best chance for success. For example, if you are also working, you may want to check with your employer to make sure that your work hours will not conflict with your course schedule. Also, schedule your class during a time of day when you are more attentive and do your best work.

This online geometry course is different from traditional math courses that you have taken in the past. You will work exercises and complete homework online. Because of this, it is your responsibility to keep a written notebook or journal of your work. You will need this documentation of your work when it comes to studying for a quiz, exam, or test.

On the day of your first class period, double-check your schedule and allow yourself plenty of time to arrive. Make sure you bring a notebook or binder, paper, and a pencil or some other writing instrument. If you are required to have a lab manual, graph paper, calculator, or some other supply, bring these with you also.

OBJECTIVE 2 ▶ General Tips for Success. Below are some general tips that will increase your chance for success in a mathematics class. Many of these tips will also help you in other courses you may be taking.

Note: Many tips have to do with the specifics of this online course and will be listed in Objective 3.

Exchange names and phone numbers with at least one other person in class. This contact person can be a great help if you miss an assignment or want to discuss math concepts or exercises that you find difficult.

Choose to attend all class periods and be on time. If possible, sit near the front of the classroom. This way, you will see and hear the presentation better. It may also be easier for you to participate in classroom activities.

Do your homework. You've probably heard the phrase "practice makes perfect" in relation to music and sports. It also applies to mathematics. You will find that the more time you spend solving math exercises, the easier the process becomes. In this online course, homework can be submitted as many times as you like. This means you can work and rework those exercises that you struggle with until you master them. It is a good idea to work through all homework exercises twice before the submission deadline. Also, be sure to schedule enough time to complete your assignments before the due date assigned by your teacher.

Check your work. Checking work is the same for an online course as it is for a traditional course. This is why it is imperative that you work each exercise on paper before submitting the answer. If it's on paper, you can go back, check your work, and follow your steps to ensure the answer is correct, or find any mistakes and correct them. If you can't find your mistake or if you have any questions, make sure you talk to your teacher.

Learn from your mistakes and be patient with yourself. Everyone, even your instructor, makes mistakes. (That definitely includes me—Elayn Martin-Gay.) Use your errors to learn and to become a better math student. The key is finding and understanding your errors.



Was your mistake a careless one, or did you make it because you can't read your own math writing? If so, try to work more slowly or write more neatly and make a conscious effort to carefully check your work.

Did you make a mistake because you don't understand a concept? Take the time to review the concept or ask questions to better understand it.

Did you skip too many steps? Skipping steps or trying to do too many steps mentally may lead to preventable mistakes.

Know how to get help if you need it. It's always a good idea to ask for help whenever there is something that you don't understand. One great advantage about doing homework in MyMathLab is that there is built-in "help" whenever you need it. Should you get a wrong answer, a box will appear and offer you hints for working the exercise correctly. Again, this is why it is so important to keep a record of all your work on paper so you can go back and follow the suggestions. You will have three attempts to get each exercise correct before it is marked wrong. **Remember:** Even though you are working online, your teacher is your most valuable resource for answering questions. Having a written journal of neatly worked exercises helps your teacher identify mistakes on an exercise or about a concept in general.

Organize your class materials, including homework assignments, graded quizzes and tests, and notes from your class or lab. All of these items will be valuable references throughout your course, especially when studying for upcoming tests or your final exam. Make sure you can locate these materials when you need them. An excellent way to do this is by using the Organizer, which is reviewed in Objective 3.

Read your ebook or watch the section lecture videos before class. Your course ebook is available through MyMathLab. Use this online text just as you would a printed textbook. Read the assigned section(s), and then write down any questions you may have. You will then be prepared to ask any questions in class the next day. Also, familiarizing yourself with the material before class will help you understand it much more readily when it is presented. There is also a reading assessment homework if assigned by your teacher.

Lecture videos, approximately 20 minutes in length, are available for every section of your ebook. These videos are specific to the material in the ebook and are presented by the ebook author, Elayn Martin-Gay. Watching a section video before class is another way to familiarize yourself with the material. Write down any questions you may have so that you can ask them in class. Watching a section video after class is also an excellent way to review concepts that are difficult for you.

Don't be afraid to ask questions. Teachers are not mind readers. Many times they do not know a concept is unclear until a student asks a question. You are not the only person in class with questions. Other students are normally grateful that someone has spoken up.

Turn in assignments on time. Always be aware of the schedule of assignments and due dates set by your teacher. Do not wait until the last minute to submit your work online. It is a good idea to submit your assignments 6–8 hours before the submission deadline to ensure some "cushion" time in case you have technology trouble.

When assignments are turned in online, it is extremely important for you to keep a copy of your written work. You will find it helpful to organize this work in a 3-ring binder. This way, you can refer to your written work to ask questions and can use it later to study for tests. (See the Video Organizer in Objective 3.)

OBJECTIVE 3 ► Understanding How to Use the Resources Provided in MathXL and MyMathLab. There are many helpful resources available to you through MathXL and MyMathLab. It is important that you understand these resources and know when to use them. Let's start with the resources that are available within MathXL to help you successfully complete and master the exercises in your assigned homework. When

working your homework assignments, you will find the following buttons listed on the right-hand side of the screen.

- **Help Me Solve This**—Select this resource to get guided, step-by-step help for the exercise you are working. Once you have reached the correct answer (through the help feature), you must work an additional exercise of the same type before you receive credit for having worked it correctly.
- **View an Example**—Select this resource to view a correctly worked example similar to the exercise you are working on. After viewing the example, you can go back to your original exercise and complete it on your own.
- **Textbook**—Select this resource to go to the section of the ebook where you can find exercises similar to the one you are working on.
- **Video**—Select this resource to view a video clip of Elayn Martin-Gay (your ebook author) working an exercise similar to the one you need help with. **Not all exercises have an accompanying video clip. This button will not be listed if no video clip is available.

Let's now take a moment to go over a few of the features available in MyMathLab to help you prepare for class, review outside class, organize, improve your study skills, and succeed.

- **Ebook and Videos**—You can choose to read the ebook and/or watch the videos for every section of the text. The ebook includes worked examples, helpful hints, practice exercises, and section exercises for every text section. Read the material actively, and make a note of any questions you have so that you can then ask them in class.

There are lecture **videos**, approximately 20 minutes in length, for every ebook section. Watch these videos to prepare for class, to review after class, or to help you catch up if you miss class. The videos are presented by your ebook author, Elayn Martin-Gay, so all material covered in the videos is consistent with the coverage in your ebook. Make a note of any questions you have after watching the videos so that you can ask your instructor. Your instructor may assign watching the videos as homework to prepare for class.

- **Video Organizer**—The **Video Organizer** is designed to help you take notes and work practice exercises while watching Elayn Martin-Gay's lecture series (available in MyMathLab and on DVD). All content in the Video Organizer is presented in the same order as it is presented in the videos, making it easy for students to create a course notebook and build good study habits!
 - Covers all of the video examples in order
 - Provides sample space for student to write down key definitions and rules
 - Includes "Play" and "Pause" button icons to prompt students to follow along with Elayn for some exercises while they try others on their own

The Video Organizer is available in a loose-leaf, notebook-ready format. It is also available for download in MyMathLab.

OBJECTIVE 4 ► Getting Help. If you have trouble completing assignments or understanding the mathematics, get help as soon as you need it! This tip is presented as an objective on its own because it is so important. In mathematics, usually the material presented in one section builds on your understanding of the previous section. This means that if you don't understand the concepts covered during a class period, there is

a good chance that you will not understand the concepts covered during the next class period. If this happens to you, get help as soon as you can.

Where can you get help? Many suggestions have been made in the section on where to get help, and now it is up to you to do it. Try your instructor, a tutoring center, or a math lab, or you may want to form a study group with fellow classmates. If you do decide to see your instructor or go to a tutoring center, make sure that you have a neat notebook and are ready with your questions.

OBJECTIVE 5 ► Preparing for and Take an Exam. Make sure that you allow yourself plenty of time to prepare for a test. If you think that you are a little “math anxious,” it may be that you are not preparing for a test in a way that will ensure success. The way that you prepare for a test in mathematics is important. To prepare for a test,

1. Review your previous homework assignments. You may also want to rework some of them.
2. Review any notes from class and section-level quizzes you have taken. (If this is a final exam, also review chapter tests you have taken.)
3. Practice working out exercises by completing the Chapter Review found at the end of each chapter.
4. Since homework exercises are online, you may easily work new homework exercises. If you open an already submitted homework assignment, you can get new exercises by clicking “similar exercise.” This will generate new exercises similar to the homework exercises you have already submitted. You can then work and rework exercises until you fully understand them. *Don’t stop here!*
5. It is important that you place yourself in conditions similar to test conditions to find out how you will perform. In other words, as soon as you feel that you know the material, try taking some sample tests.
In your ebook, there are two forms of chapter tests at the end of each chapter. One form is an open response test form and the second is a standardized test form. You can use these two tests as practice tests. Do not use your notes or any other help when completing these tests. Check your answers by using the answer section in the ebook. There are also exact video clip solutions to the open response practice test form. Finally, identify any concepts that you do not understand and consult your teacher.
6. Get a good night’s sleep before the exam.
7. On the day of the actual test, allow yourself plenty of time to arrive at your exam location.

When taking your test,

1. Read the directions on the test carefully.
2. Read each problem carefully as you take the test. Make sure that you answer the question asked.
3. Pace yourself by first completing the problems you are most confident with. Then work toward the problems you are least confident with. Watch your time so you do not spend too much time on one particular problem.
4. Do not turn your test in early. If you have extra time, spend it double-checking your work and answers.

OBJECTIVE 6 ► Managing Your Time. As a student, you know the demands that classes, homework, work, and family place on your time. Some days you probably wonder

how you'll ever get everything done. One key to managing your time is developing a schedule. Here are some hints for making a schedule:

1. Make a list of all of your weekly commitments for the term. Include classes, work, regular meetings, extracurricular activities, etc.
2. Next, estimate the time needed for each item on the list. Also make a note of how often you will need to do each item. Don't forget to include time estimates for the reading, studying, and homework you do outside of your classes. You may want to ask your instructor for help estimating the time needed.
3. In the following exercise set, you are asked to block out a typical week on the schedule grid given. Start with items with fixed time slots like classes and work.
4. Next, include the items on your list with flexible time slots. Think carefully about how best to schedule items such as study time.
5. Don't fill up every time slot on the schedule. Remember that you need to allow time for eating, sleeping, and relaxing! You should also allow a little extra time in case some items take longer than planned.
6. If you find that your weekly schedule is too full for you to handle, you may need to make some changes in your workload, classload, or in other areas of your life. If you work, you may want to talk to your advisor, manager or supervisor, or someone in your school's counseling center for help with such decisions.

1.1 | EXERCISE SET MyMathLab®

1. How many times is it suggested that you work through homework exercises before the submission deadline?
2. How does the "Help Me Solve This" feature work?
3. Why is it important that you write your step-by-step solutions to homework exercises and keep a hard copy of all work submitted online?
4. How many times are you allowed to submit homework online?
5. How can the lecture videos for each section help you in this course? When is the best time to use them?
6. In the homework assignments, how many attempts do you get to correct an exercise before it is marked incorrect?
7. If the "View an Example" feature is used, is it necessary to work an additional exercise before continuing the assignment?
8. How does reading the ebook section before class help you prepare for class?
9. Do all homework exercises in MyMathLab come with an accompanying video clip solution?
10. How can you use MathXL to contact your teacher about an exercise you don't understand?
11. When are your homework assignments due?
12. How much "cushion" time is recommended before your deadline when submitting homework online?
13. Is it still OK to ask your teacher for help even though this is an online course?
14. Name two ways you can prepare for any chapter tests.
15. If you are absent, name two ways you can review the material you missed.
16. List the resources available in MyMathLab to help you. Which of these resources do you think will be most helpful to you?
17. Are you allowed to use a calculator in this class?
18. Review Objective 6 and fill in the schedule grid that follows.
19. Study your completed grid from Exercise 18. Decide whether you have the time necessary to successfully complete this course and any others you have scheduled.

	<i>Monday</i>	<i>Tuesday</i>	<i>Wednesday</i>	<i>Thursday</i>	<i>Friday</i>	<i>Saturday</i>	<i>Sunday</i>
4:00 a.m.							
5:00 a.m.							
6:00 a.m.							
7:00 a.m.							
8:00 a.m.							
9:00 a.m.							
10:00 a.m.							
11:00 a.m.							
12:00 p.m.							
1:00 p.m.							
2:00 p.m.							
3:00 p.m.							
4:00 p.m.							
5:00 p.m.							
6:00 p.m.							
7:00 p.m.							
8:00 p.m.							
9:00 p.m.							
10:00 p.m.							
11:00 p.m.							
Midnight							
1:00 a.m.							
2:00 a.m.							
3:00 a.m.							

1.2 GEOMETRY—A MATHEMATICAL SYSTEM

OBJECTIVES

- 1 Use Logic to Recognize Patterns.
- 2 Understand How a Mathematical System, Like Geometry, Is Formed.

Helpful Hint

To be more specific, “geo” means earth and “metron” means measurement.

OBJECTIVE 1 ▶ Using Logic to Recognize Patterns. The word **geometry** comes from the Greek language and means to measure the earth. (For more on the origin of geometry, see the Chapter 1 Opener, page 1.)

We use geometry in many different situations. It is still needed for measuring the perimeter of a backyard for the purpose of constructing a fence, or for calculating the area of a backyard for the purpose of ordering grass seed. Nowadays, the use of geometry is all around you, and some applications can easily be seen, especially in architecture and construction. Look at the angles of a building—the windows, the roof, any brick or stone placement, etc. The next time you approach a bridge, notice the engineering and geometry involved in constructing the bridge. More recently, geometry is greatly used in the construction of cell phones and cell towers, GPS systems, and in the development of 3-D movies.

VOCABULARY

- geometry
- logic
- undefined term
- defined term
- postulate
- axiom
- theorem



Perimeter needed for fencing.
Area of yard needed for grass seed.



Notice the angles in this building.

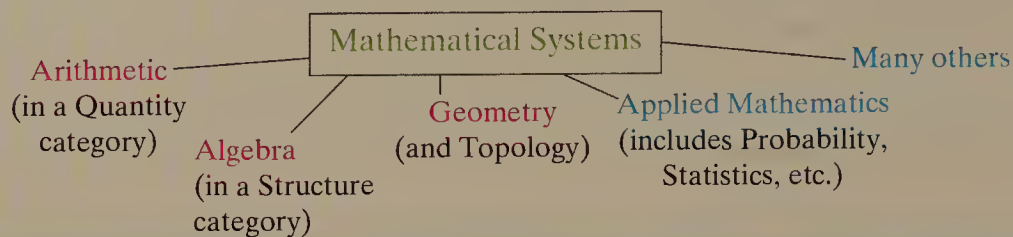


Ravenel Bridge near Charleston, SC

Helpful Hint

The number of mathematical systems has become so large, that there is a Mathematics Classification System. It is used by authors of research papers to clearly classify the subject of their paper.

Below, we take great liberty and diagram a few of the major divisions that you may be familiar with.



Before we talk more about geometry and mathematical systems, let's check our reasoning skills, or **logic**. We need to learn how to use reason and logic in order to recognize patterns, make conjectures, and develop the laws of this system. Let's start to examine the mathematical system of geometry by first recognizing patterns and making conjectures. Later in this text, we will discuss reasoning further, including different types of reasoning.

We can use logic to look for patterns in figures, as shown in Example 1.

EXAMPLE 1 Using Logic to Recognize Patterns

Sketch the next figure in this pattern.



Solution From left to right, each figure is a square, with a triangle shaded in a different corner. From left to right, the shading is moving clockwise. Thus, we predict that the next figure is

**PRACTICE****1**

Sketch the next figure in the pattern.



Patterns can also appear in lists of numbers.

EXAMPLE 2 Using Logic to Recognize Patterns

Look for a pattern and predict the next number.

a. 2, 10, 50, 250, ...

b. -3, 0, 3, 6, 9, ...

Solution a. 2, 10, 50, 250
 $(\times 5) (\times 5) (\times 5)$

Each number is 5 times the previous number. We predict the next number to be 1250 because

$$250 \times 5 = 1250$$

- b. Each number is 3 more than the previous number. We predict the next number to be 12 because

$$9 + 3 = 12$$

□

PRACTICE

2

Look for a pattern and predict the next number.

a. $-1, 1, -2, 2, -3, \dots$

b. $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \dots$

OBJECTIVE 2 ▶ Understanding How a Mathematical System Is Formed. Now we can use our logic and reasoning skills to develop the mathematical system of geometry. How do we begin?

We begin with **undefined terms**, which we first describe.

Then we use these undefined terms to formally define terms. From these, we use our **defined terms** to write statements that we do not prove, but instead agree and accept them to be true. These statements are called **postulates** or **axioms**. Our mathematical system grows by using terms, postulates, and axioms to prove **theorems**.

It continues to grow by using theorems to prove other theorems.

▶ Helpful Hint

What is the difference between a postulate and an axiom? Although we may use these words interchangeably, the table below may help:

	<i>Axiom</i>	<i>Postulate</i>
Similar	Do <i>not</i> prove these statements	
Possible	Thought of as a global,	A truth that is more
Differences	self-evident truth or common notion	specific to geometry

A Mathematical System consists of

Undefined terms are the most basic of terms that we do not formally define, but have a meaning or description that we agree upon.

To form **definitions**, we use undefined terms or already defined terms to formally define them.

Postulates (or Axioms) are statements that we accept as true and do not try to prove.

Theorems are statements that we prove using logic, the foundation above, and other previously proved theorems.

The
Foundation

Use the
Foundation
for:

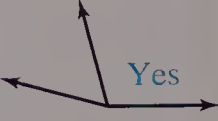
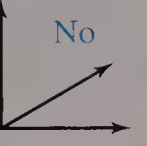
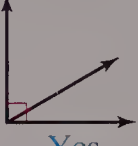
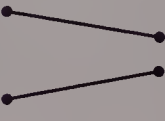
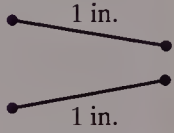
Some examples of undefined terms are *point*, *line*, *plane*, and the idea of *between*. For example, what does it mean for a point on a line to be *between* two other points?

In the next section, we begin to develop our mathematical system called geometry.

Sometimes in geometry we may be faced with unmarked figures. If this occurs, we must be careful when making assumptions about these figures. For example, let's look at the following figures and learn the assumptions that we may or may not make.

Figure Assumptions

Be careful when assuming anything from a marked or unmarked figure.
Examples: Can you assume that ...

• This is a line?  Yes	• This is a right angle (measures 90°)?  No	• This is a right angle?  Yes	• These angles are supplementary (sum of 180°)?  Yes	• These angles have a common side?  Yes
• These two lines intersect to form angles?  Yes	• These angles are complementary (sum of 90°)?  No	• These angles are complementary (sum of 90°)?  Yes	• These segments have equal length?  No	• These segments have equal length?  Yes

In Example 3 and associated exercises, we assume a basic knowledge of squares, triangles, rectangles, and some angle measures.

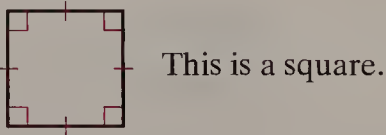
EXAMPLE 3 Given the figure, can you conclude the statement?



Solution No, we cannot assume that the figure is a square. A square has 4 right angles and 4 sides of equal length. Correctly marked,

Helpful Hint

The same number of tick marks on sides indicates equal length.



PRACTICE 3

Given the figure, can you conclude the statement?



VOCABULARY & READINESS CHECK

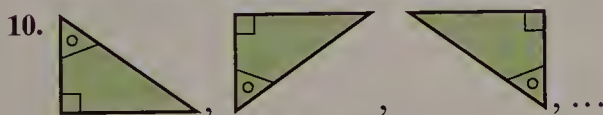
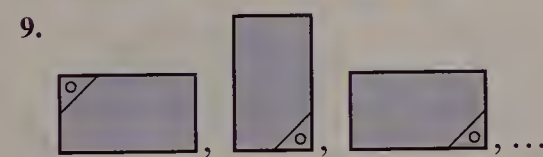
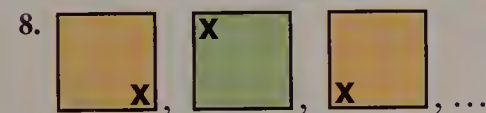
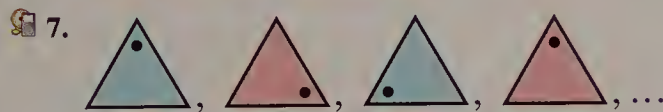
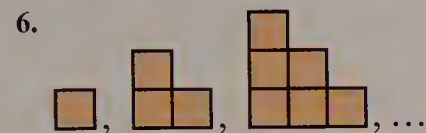
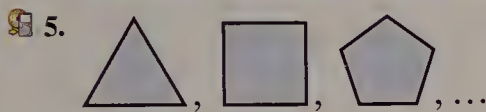
Word Bank Use the choices to fill in each blank. Some choices may be used more than once and some not at all.

- | | | |
|----------------|--------------|---------|
| undefined term | axiom | theorem |
| postulate | defined term | |

1. Which word or phrase above describes a word that has been defined? _____
2. Which word or phrase above describes a word that is described, but not defined? _____
3. A statement that we prove is called a(n) _____.
4. A statement that we accept as true but do not prove is called a(n) _____ or a(n) _____.

1.2 EXERCISE SET MyMathLab®

Look for a pattern and sketch the next figure. See Example 1.



Look for a pattern and predict the next two numbers. See Example 2.

11. $-10, 0, 10, 20, \dots$ 12. $-6, -4, -2, 0, \dots$

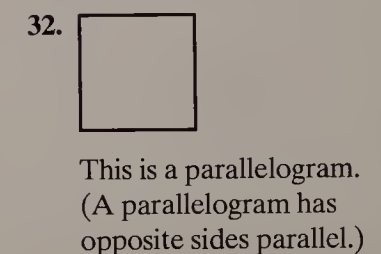
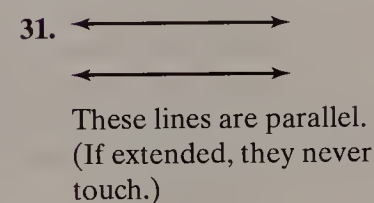
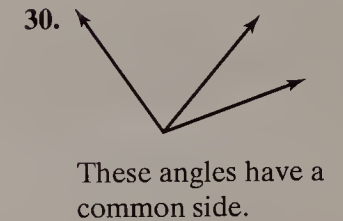
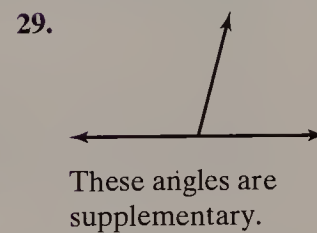
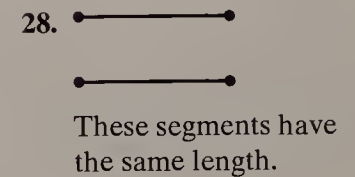
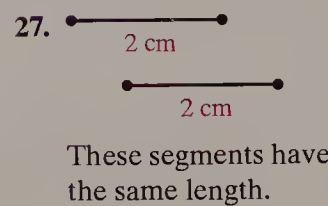
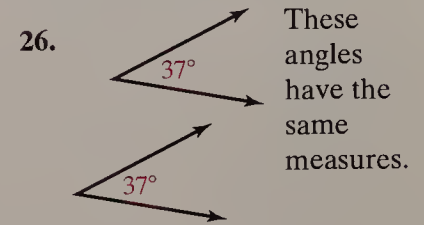
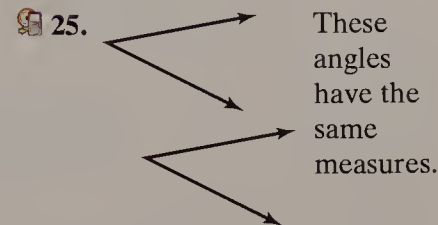
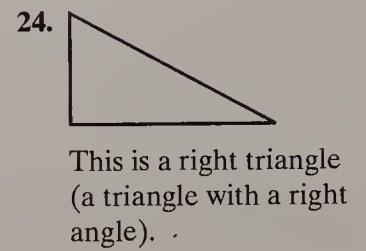
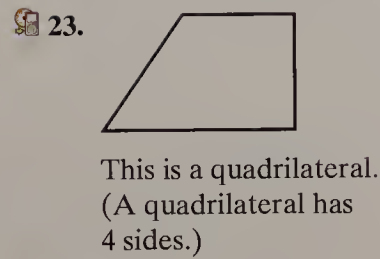
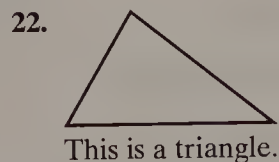
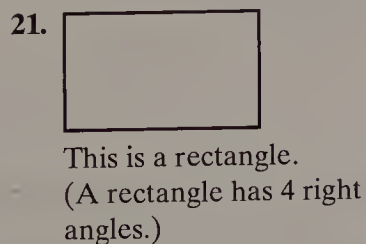
13. $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots$ 14. $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots$

15. $1, 4, 16, 64, \dots$ 16. $2, 6, 18, 54, \dots$

17. $\frac{1}{2}, \frac{1}{10}, \frac{1}{50}, \frac{1}{250}, \dots$ 18. $\frac{1}{5}, \frac{1}{10}, \frac{1}{20}, \dots$

19. $-7, 0, 7, 0, -7, \dots$ 20. $-1, 0, 1, 0, -1, \dots$

Given each figure, conclude whether or not each statement is true. Answer yes or no. See the Figure Assumptions in this section and Example 3.

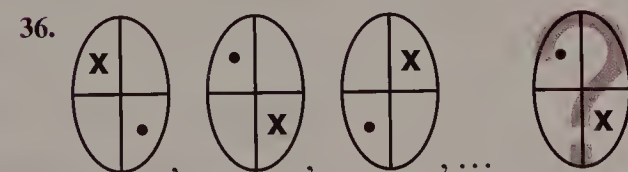


CONCEPT EXTENSIONS

Find the Error A fellow student shows you his/her next number or figure in a list. Decide whether this next number or figure is correct. If not correct, tell why and provide the correct one.

33. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \dots, \frac{1}{64}$ Square the denominator to get the next denominator.

34. $\frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \dots, \frac{1}{12}$ Double the denominator to get the next denominator.



37. Describe a difference between a postulate and a theorem.

38. Describe a similarity between a postulate and an axiom.

1.3 POINTS, LINES, AND PLANES

OBJECTIVE

- 1 Learn the Basic Terms and Postulates of Geometry.

VOCABULARY

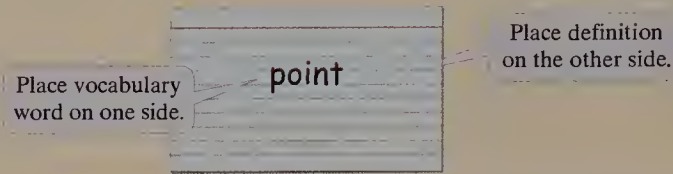
- point
- line
- plane
- lie on
- collinear
- coplanar
- space
- geometric figure
- between
- segment or line segment
- ray
- opposite rays
- intersection

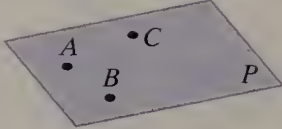
OBJECTIVE 1 ▶ Learning Terms and Postulates of Geometry. Let’s begin to examine the mathematical system of geometry as we should—with the undefined terms of point, line, and plane. These are described after the Helpful Hint.

▶ Helpful Hint

This is the beginning of the language of geometry. It is very important to study and understand ALL of the new vocabulary words and phrases.

You may want to start your own index card listing of vocabulary words.



Undefined Terms		
Description	How to Name It	Example
A point has no dimension (no length, width, or height). It does have a location, or position.	Name a point by a single capital letter.	 or Point A
A line extends in opposite directions without end and has one dimension—length.	Name a line by a single lowercase letter or by any two points on the line.	 Line ℓ or $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$
A plane extends in two dimensions without end. The two dimensions are length and width, but no thickness. We represent a plane by a flat surface.	Name a plane by a single capital letter or by any three points on the plane (that do not lie on the same line).	 Plane P or plane ABC

▶ Helpful Hint

In this text:

- A line is a straight line.
- A line contains an infinite number of points.
- A plane contains an infinite number of lines.
- Our drawings of planes appear to end and have edges, but we must remember that planes extend without end.

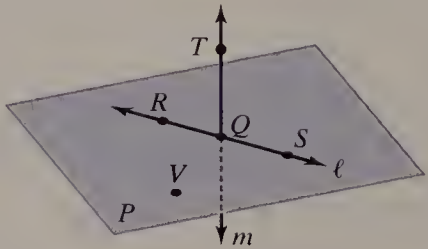
Now we use our undefined terms to write a few formal definitions. The phrase **lie on** is used in both definitions below. This is an undefined concept that is assumed to be commonly understood.

Collinear points are points that lie on the same *line*.

Coplanar points are points that lie on the same *plane*.

EXAMPLE 1 Naming Points, Lines, and Planes

- Write two other ways to name $\overleftrightarrow{QT}$.
- Write two other ways to name plane P.
- Name three points that are collinear.
- Name four points that are coplanar.
- Name three points that are not collinear.

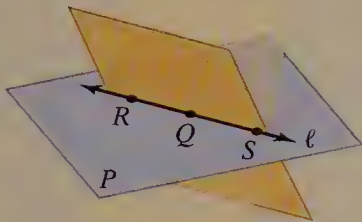


Solution

- Two other ways to name $\overleftrightarrow{QT}$ are $\overleftrightarrow{TQ}$ and line m .
- There are many correct answers. Two other ways to name plane P are plane RQV and plane RSV.
- Points R, Q, and S lie on the same line, so they are collinear.

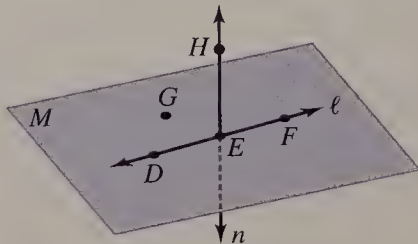
Helpful Hint

Do *not* use collinear points to name a plane since many planes exist that go through a single line.



PRACTICE 1

- a. Write two other ways to name $\overleftrightarrow{EH}$.
- b. Write two other ways to name plane M .
- c. Name three points that are collinear.
- d. Name four points that are coplanar.
- e. Name three points that are not collinear.



Let's continue to use our undefined terms to define another term.

a collection of objects

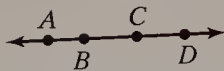
Space is the set of all points in three dimensions.

A **geometric figure** is any nonempty subset of space.

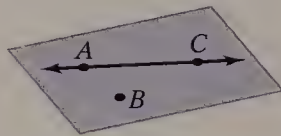
means that a geometric figure contains points

As mentioned earlier, the idea of between is another undefined term or concept. We have a basic idea of what it means for a point on a line to be between two other points on a line.

Use the term **between** when points are on the same line (or line segment).



Point B is between points A and D .
Point B is between points A and C .



Point B is NOT between points A and C .

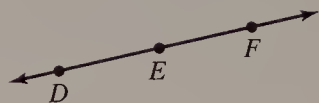
Defined Terms

Definition	How to Name It	Example
A line segment or simply segment is part of a line. It consists of two endpoints and all the points between them.	Name a segment by its end points: $\overline{AB}$ (segment AB) or $\overline{BA}$ (segment BA)	segment $\overline{AB}$ or $\overline{BA}$
A ray is part of a line. It consists of an endpoint and all points of a line on one side of the endpoint.	Name a ray by its endpoint and any other point on the ray. Here, the order of points is important—list the endpoint first. $\overrightarrow{AB}$ (ray AB)	ray $\overrightarrow{AB}$ ray $\overrightarrow{BA}$
Opposite rays are two rays that share the same endpoint and form a line.	Name each opposite ray as you would name a ray.	opposite rays $\overrightarrow{CB}$ and $\overrightarrow{CA}$

EXAMPLE 2 Naming Segments and Rays

Use the given figure to find the following.

- a. Name the segments in the figure.
- b. Name the rays in the figure.
- c. Which of the rays in part **b** are opposite rays?



Helpful Hint

Notice that by our description on the previous page, the simplest geometric figure is a point.

Helpful Hint

- Rays: Take care when naming rays. For example, $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are *not* the same ray, $\overrightarrow{AB}$ has endpoint A and $\overrightarrow{BA}$ has endpoint B .
- Lines: $\overleftrightarrow{AB}$ and $\overleftrightarrow{BA}$ name the same line.
- Segments: $\overline{AB}$ and $\overline{BA}$ name the same segment.

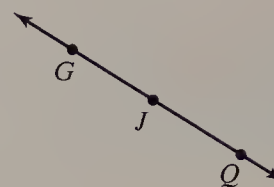
Solution

- The three segments are $\overline{DE}$ or $\overline{ED}$, $\overline{EF}$ or $\overline{FE}$, and $\overline{DF}$ or $\overline{FD}$.
- The four rays are $\overrightarrow{DE}$ or $\overrightarrow{DF}$, $\overrightarrow{ED}$, $\overrightarrow{EF}$, and $\overrightarrow{FD}$ or $\overrightarrow{FE}$.
- The opposite rays are $\overrightarrow{ED}$ and $\overrightarrow{EF}$.

□

PRACTICE**2** Use the given figure to find the following.

- Name the segments.
- Name the rays.
- Which of the rays in part b are opposite rays?

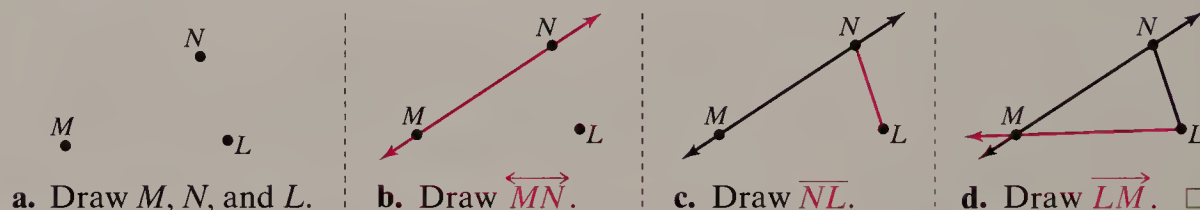


Once we know how to name terms, we can draw them and use our proper labeling, as in Example 3.

EXAMPLE 3 Drawing Lines, Segments, and Rays

Perform parts **a** through **d** in order to create a single figure.

- Draw three noncollinear points, M , N , and L .
- Draw $\overleftrightarrow{MN}$.
- Draw $\overline{NL}$.
- Draw $\overrightarrow{LM}$.

Solution**PRACTICE****3** Perform parts **a** through **d** in order to create a single figure.

- Draw three noncollinear points, A , B , and C .
- Draw $\overleftrightarrow{BC}$.
- Draw $\overleftrightarrow{CA}$.
- Draw $\overline{AB}$.

Recall that a **postulate** or **axiom** is a statement accepted without proof.

We use some of the following geometry postulates in algebra. For example, Postulate 1.3-1 is used to graph equations such as $y = 2x + 8$. We graph two points that are solutions to the equation and draw the line through the points.

Postulate 1.3-1 Two Points Determine a Line

Through any two points there is exactly one line.

Line t passes through points A and B . Line t is the only line that passes through both points.

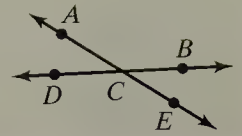


When we have two or more geometric figures, their **intersection** is the set of points that the figures have in common.

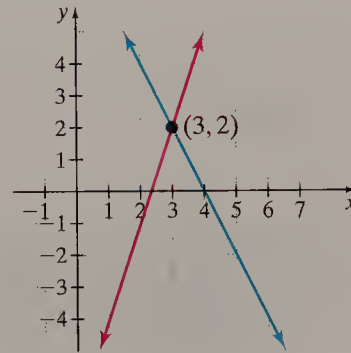
Postulate 1.3-2 Intersection of Lines

If two distinct lines intersect, then they intersect at exactly one point.

$\overleftrightarrow{AE}$ and $\overleftrightarrow{DB}$ intersect at point C .



Here is an example of the point of intersection of two lines on the coordinate plane.



Let's make sure we can name opposite rays.

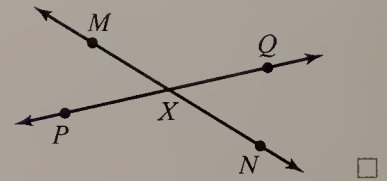
EXAMPLE 4 Opposite Rays

Name two pairs of opposite rays.

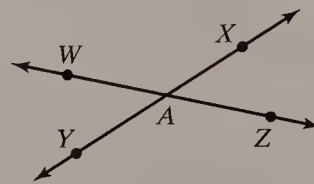
Solution

Points M , N , and X are collinear and X is between M and N . So, $\overrightarrow{XM}$ and $\overrightarrow{XN}$ are opposite rays.

Points P , Q , and X are collinear and X is between P and Q . So, $\overrightarrow{XP}$ and $\overrightarrow{XQ}$ are opposite rays.

**PRACTICE**

4 Name two pairs of opposite rays.

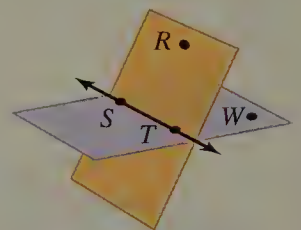


Postulate 1.3-2 in this section has to do with the intersection of lines. Now let's introduce a postulate that has to do with the intersection of planes.

Postulate 1.3-3 Intersection of Planes

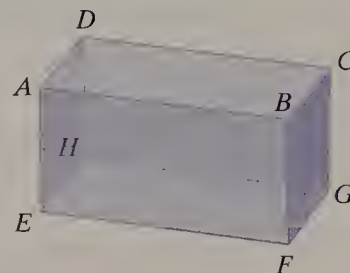
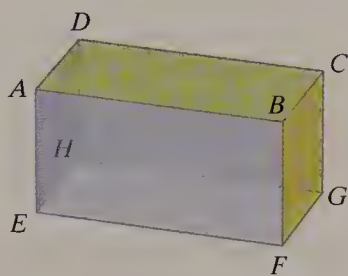
If two distinct planes intersect, then they intersect in exactly one line.

Plane RST and plane WST intersect in $\overleftrightarrow{ST}$.

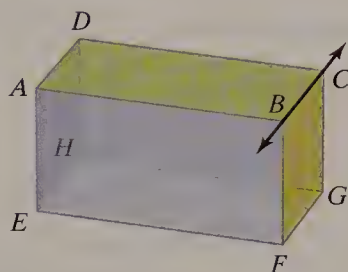


EXAMPLE 5 Finding the Intersection of Two Planes

Each surface of the box shown represents part of a plane.
What is the intersection of plane ADC and plane BFG ?

**Solution**

Shade plane ADC and plane BFG to see where they intersect. Remember from Postulate 1.3-3 that two distinct planes intersect in a line.



We can see that both planes contain point B and point C . These points define a distinct line.

The planes intersect in $\overleftrightarrow{BC}$.

PRACTICE

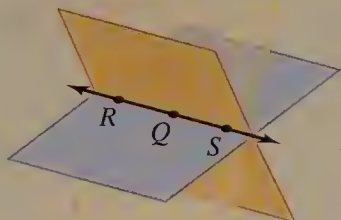
5 Use the figure for Example 5. What is the intersection of plane ADH and plane EFG ?

Photographers use three-legged tripods to make sure that a camera is steady. The feet of the tripod all touch the floor at the same time. You can think of the feet as points and the floor as a plane. As long as the feet do not all lie in one line, they will lie in exactly one plane.

This illustrates Postulate 1.3-4.

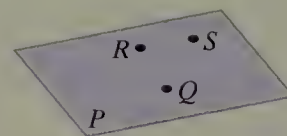
**Helpful Hint**

Don't forget that the points *must* be noncollinear. If they are collinear, many planes pass through them.

**Postulate 1.3-4** Three Noncollinear Points Determine a Plane

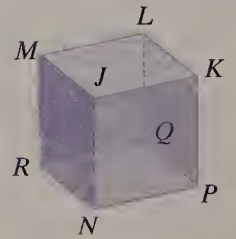
Through any three noncollinear points there is exactly one plane.

Points Q , R , and S are noncollinear. Plane P is the only plane that contains them.



EXAMPLE 6 Using Postulate 1.3-4

Use the figure shown to answer the question.
What plane contains points N , P , and Q ? Shade the plane.



Solution



The plane on the bottom of the figure contains points N , P , and Q .

PRACTICE

6 What plane contains points J , M , and Q ? Shade the plane. (Hint: The plane does not have to be already drawn in the figure.)

VOCABULARY & READINESS CHECK

Matching Match each symbol with its meaning.

- | | | | |
|--------------------|------------------------------|--------------------------|------------------------------|
| 1. $\overline{RS}$ | 2. $\overleftrightarrow{RS}$ | 3. $\overrightarrow{RS}$ | 4. $\overleftrightarrow{SR}$ |
| A. line RS | B. ray SR | C. segment RS | D. ray RS |

Word Bank Use the choices to fill in each blank.

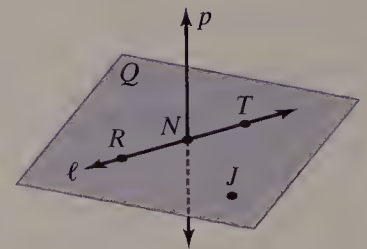
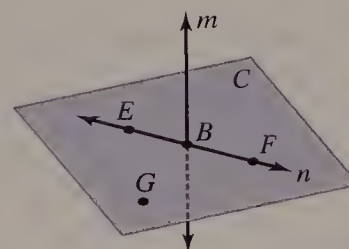
point	collinear	segment	opposite rays
line	coplanar	ray	geometric figure
plane	space		

- A(n) _____ extends in opposite directions without end.
- A(n) _____ extends in two dimensions without end.
- A(n) _____ consists of an endpoint and all points of a line on one side of the endpoint.
- A(n) _____ consists of two endpoints and all the points between them.
- _____ points lie on the same plane.
- _____ points lie on the same line.
- A(n) _____ is any nonempty subset of space.
- _____ are two rays that share an endpoint and form a line.
- A(n) _____ has a location but no dimension.
- _____ is the set of all points.

1.3 EXERCISE SET MyMathLab®

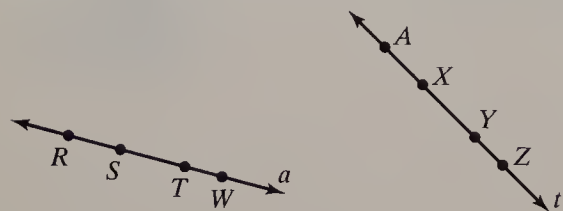
Use the figures at the right for Exercises 1–8. See Example 1.

- What are two other ways to name $\overleftrightarrow{EF}$?
- What are two other ways to name $\overleftrightarrow{RT}$?
- What are two other ways to name plane C ?
- What are two other ways to name plane Q ?
-  Name three collinear points on plane C .
- Name three collinear points on plane Q .



7. Name four coplanar points on plane C .
8. Name four coplanar points on plane Q .

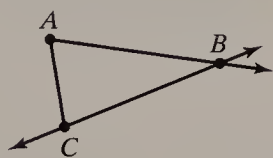
Use the figures below for Exercises 9–14. See Example 2.



9. Name the segments on line a .
10. Name the segments on line t .
11. Name the rays on line a .
12. Name the rays on line t .
13. Name the pair of opposite rays with endpoint T .
14. Name the pair of opposite rays with endpoint X .

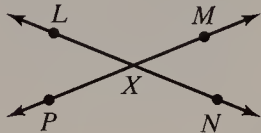
Use the figure at the right for Exercises 15–18. See Examples 2 and 3.

15. Name three points.
16. Name the line(s).
17. Name the line segment(s).
18. Name the ray(s).



True or False Use the figure at the right for Exercises 19–22. See Example 4.

19. $\overrightarrow{XL}$ and $\overrightarrow{LX}$ are opposite rays.
20. $\overrightarrow{XP}$ and $\overrightarrow{XN}$ are opposite rays.
21. $\overrightarrow{XM}$ and $\overrightarrow{XP}$ are opposite rays.
22. $\overrightarrow{XL}$ and $\overrightarrow{XN}$ are opposite rays.



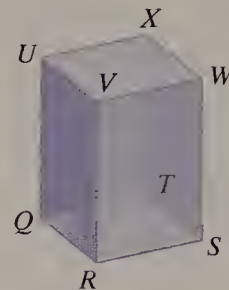
Mixed Practice See Examples 1–4. For Exercises 23–26, name three points in each diagram that are not collinear.

- 23.
- 24.
- 25.
- 26.

True or False Use the figure for Exercise 24 to answer each question either true or false.

27. Point A lies on line l .
28. Point D lies on line m .
29. Point B lies on line l .
30. Point C lies on line m .
31. A , B , and C are coplanar.
32. D , E , and B are collinear.
33. A , B , and C are collinear.
34. D , E , and B together lie on a single plane only.

Use the figure at the right for Exercises 35–42. See Example 5. Name the intersection of each pair of planes.



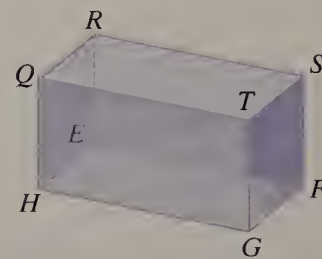
35. planes QRS and RSW
36. planes UXV and WVS
37. planes XWV and UVR
38. planes TXW and TQU

Name two planes that intersect in the given line.

39. $\overleftrightarrow{QU}$
40. $\overleftrightarrow{TS}$
41. $\overleftrightarrow{XT}$
42. $\overleftrightarrow{VW}$

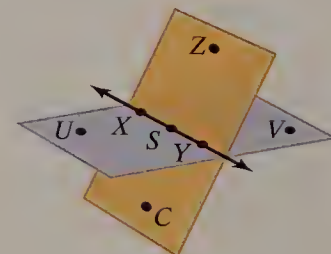
Name a point that is coplanar with the given points. See Example 6.

43. Q , R , and S
44. T , S , and F
45. G , Q , and T
46. E , F , and G
47. Q , R , and H
48. R , S , and F
49. Q , R , and F
50. R , S , and G



Postulate 1.3-4 states that any three noncollinear points lie in exactly one plane. Find the plane that contains the first three points listed. Then determine whether the fourth point is in that plane. Write “coplanar” or “noncoplanar” to describe the points. See Example 6.

51. Z , S , Y , C
52. S , U , V , Y
53. X , Y , Z , U
54. X , S , V , U
55. X , Z , S , V
56. S , V , C , Y



Sketch Sketch the figure described. Assume that each figure lies on a plane.

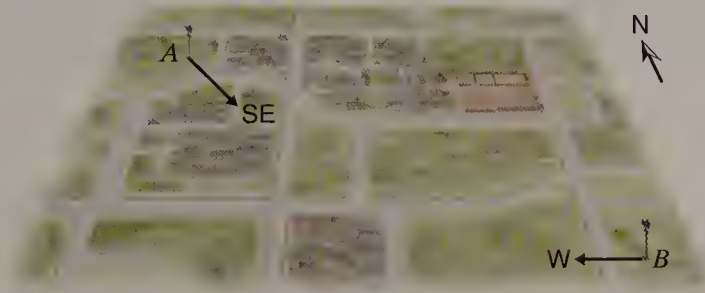
57. Two lines that do not intersect.
58. Three lines that do not intersect.
59. Three lines that intersect at a point.
60. Four lines that intersect at a point.
61. Two lines that intersect at point A and another line that intersects both lines, but not at point A .
62. Three lines that intersect at point A and another line that intersects all three lines, but not at point A .

CONCEPT EXTENSIONS

For Exercises 63–68, determine whether each statement is “always,” “sometimes,” or “never” true.

63. $\overleftrightarrow{TQ}$ and $\overleftrightarrow{QT}$ are the same line.
64. $\overrightarrow{JK}$ and $\overrightarrow{JL}$ are the same ray.
65. Two distinct intersecting lines are coplanar.
66. Four points are coplanar.
67. A plane containing two points of a line contains the entire line.
68. Two distinct lines intersect in more than one point.

69. A cell phone tower at point A receives a cell phone signal from the southeast. A cell phone tower at point B receives a signal from the same cell phone from due west. Trace the diagram below and find the location of the cell phone. Describe how Postulates 1.3-1 and 1.3-2 help you locate the phone.



70. You can represent the hands on a clock at 6:00 as opposite rays. Estimate the other 11 times on a clock that you can represent as opposite rays. (It may help to sketch the face of a clock.)

Coordinate Geometry Graph the points on a coordinate plane and state whether they appear to be collinear.

71. $(1, 1), (4, 4), (-3, -3)$

72. $(2, 4), (4, 6), (0, 2)$

73. $(0, 0), (-5, 1), (6, -2)$

74. $(0, 0), (8, 10), (4, 6)$

75. $(0, 0), (0, 3), (0, -10)$

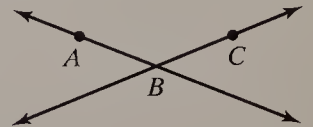
76. $(-2, -6), (1, -2), (4, 1)$

77. How many planes contain the same three collinear points? Explain.

78. How many planes contain a given line? Explain.

79. Suppose two points are in plane P . Explain why the line containing the points is also in plane P .

80. Suppose two lines intersect. How many planes do you think contain both lines? Use the diagram at the right and your answer to Exercise 79 to explain your answer.



1.4 SEGMENTS AND THEIR MEASURE

OBJECTIVES

- 1 Understand the Measure of Segments.
- 2 Use Segment Postulates and Algebra to Find Segment Lengths.

VOCABULARY

- coordinate
- distance
- congruent segments
- midpoint
- bisect
- segment bisector

Helpful Hint

Be very careful with notation.
 $\overline{AB}$ means line segment AB .
 AB means the length of $\overline{AB}$.

OBJECTIVE 1 Understanding Segment Measures. In the previous section, we defined many terms and stated a few postulates. Now, let's expand our geometry system by measuring segments. To measure a segment, let's learn how to find the distance between two points on a segment.

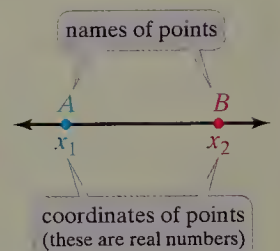
Postulate 1.4-1 Ruler Postulate

The points on a line can be paired, one-to-one, with a real number. The real number that corresponds to a point is called the **coordinate** of the point.

The above allows us to calculate distance. The **distance** between points A and B is the absolute value of the difference of their coordinates.

We use the notation AB for distance or the length of $\overline{AB}$.

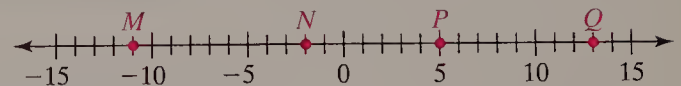
$$AB = |x_2 - x_1|$$



In Example 1, we find the distance between two points on a number line.

EXAMPLE 1 Measuring Lengths of Segments

Find MP .



Solution Recall that MP is the distance between points M and P . Use the Ruler Postulate to find MP .

The coordinate of M is -11 . The coordinate of P is 5 .

$$MP = |5 - (-11)| = |16| = 16 \text{ or } MP = |-11 - 5| = |-16| = 16$$

Thus, the distance is 16 units. □

PRACTICE

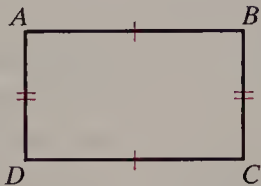
- 1 Find NQ .

Helpful Hint
Segments are congruent, $\cong$.
Lengths are equal, $=$.

Now that we can measure segments, we can talk about the notation used for segments of equal ($=$) length. Two segments that have the same length are called **congruent segments**. The symbol $\cong$ means congruent.

Congruent Segments vs. Equal Lengths
If $\overline{AB} \cong \overline{CD}$, then $AB = CD$.
Also, if $AB = CD$, then $\overline{AB} \cong \overline{CD}$.

We mark congruent segments in a figure with exactly the same number of tick marks.



Note: $\overline{AB}$ and $\overline{DC}$ both have 1 tick mark, so $\overline{AB} \cong \overline{DC}$.
 $\overline{AD}$ and $\overline{BC}$ both have 2 tick marks, so $\overline{AD} \cong \overline{BC}$.

EXAMPLE 2 Measuring Congruent Segments

- a. Use the figure above to write the congruent segments and the equal distances.
- b. If $BC = 2$ feet, find AD .
- c. If $DC = 6$ feet, find AB .

Solution a. $\overline{AD}$ and $\overline{BC}$ are marked the same, so $\overline{AD} \cong \overline{BC}$ and $AD = BC$.



Also, $\overline{AB}$ and $\overline{DC}$ are marked the same, so $\overline{AB} \cong \overline{DC}$ and $AB = DC$.

- b. $BC = AD$, so if $BC = 2$ feet, then $AD = 2$ feet.
- c. $DC = AB$, so if $DC = 6$ feet, then $AB = 6$ feet.

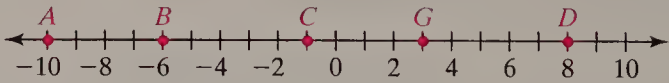
PRACTICE

- 2** a. Use the figures below to write the congruent segments and the equal distances.
b. If $AC = 10$ cm, find DF .
c. If $CB = 25$ cm, find FE .



EXAMPLE 3 Determining if Segments Are Congruent

Given the figure, is $\overline{BG} \cong \overline{AC}$?



Solution Remember: If $BG = AC$, then $\overline{BG} \cong \overline{AC}$.

$BG = |3 - (-6)| = |9| = 9,$
 $AC = |-1 - (-10)| = |-1 + 10| = |9| = 9$

Since $BG = AC$, then $\overline{BG} \cong \overline{AC}$.
distances equal segments congruent

PRACTICE

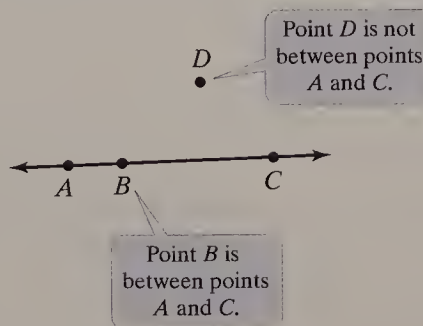
3 Using the figure for Example 3, is $\overline{BC} \cong \overline{GD}$?

Answer yes or no.

Remember that **between** is an undefined term. If we have three distinct points on a line, then one of the points will be between the other two.

Helpful Hint

To use the concept of betweenness, the points *must* lie on the same line (be collinear).



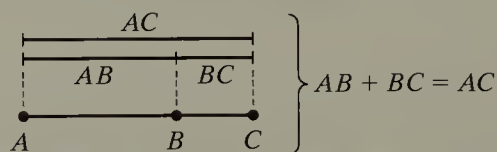
We can use the concept of betweenness to add the lengths of segments.

Helpful Hint

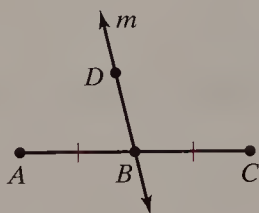
Since AB , BC , and AC are real numbers, then segment subtraction is possible also. We review this in Section 2.6.

Postulate 1.4-2 Segment Addition Postulate

If point B is between points A and C , then $AB + BC = AC$. Also, if $AB + BC = AC$, then point B is between points A and C .



The **midpoint** of a segment is a point that divides, or **bisects**, a segment into two congruent segments.



True statements:

- B is the midpoint of $\overline{AC}$.
- Line m bisects $\overline{AC}$.
- $\overleftrightarrow{BD}$ bisects $\overline{AC}$.
- $\overline{BD}$ bisects $\overline{AC}$.
- $\overline{AB} \cong \overline{BC}$.
- $AB = BC$.

Helpful Hint

A segment bisector contains the midpoint of a segment, but no other point of the segment.

A line, ray, segment, or plane that intersects a segment at its midpoint is called a **segment bisector**.

OBJECTIVE 2 Using Algebra to Find Segment Lengths.

EXAMPLE 4 Using the Segment Addition Postulate

If $MP = 47$ units, find MN and NP .



Solution Since we know MP , let's use the Segment Addition Postulate and algebra to first find x .

$$\begin{array}{rcl}
 MN + NP = MP & \text{Segment Addition Postulate} \\
 \swarrow \quad \searrow \quad \searrow & & \\
 (5x + 1) + (11x - 2) = 47 & \text{Substitution} \\
 16x - 1 = 47 & \text{Combine like terms.} \\
 16x = 48 & \text{Add 1 to both sides.} \\
 \frac{16x}{16} = \frac{48}{16} & \text{Divide both sides by 16.} \\
 x = 3 & \text{Simplify.}
 \end{array}$$

Now use the value of x to find MN and NP .

$$\left. \begin{array}{l} MN = 5x + 1 = 5(3) + 1 = 16 \\ NP = 11x - 2 = 11(3) - 2 = 31 \end{array} \right\} \text{Let } x = 3.$$

Check: See that $MP = 47$.

$$\begin{array}{c}
 \begin{array}{ccccc}
 & 5x + 1 & & 11x - 2 & \\
 \bullet & & \bullet & & \bullet \\
 M & & N & & P
 \end{array} \\
 \text{or} \\
 16 \quad + \quad 31 = 47 \quad \text{Check.}
 \end{array}$$

PRACTICE

4 If $QS = 79$, find QR and RS .

$$\begin{array}{ccccc}
 & 12x - 3 & & 8x + 2 & \\
 \bullet & & \bullet & & \bullet \\
 Q & & R & & S
 \end{array}$$

EXAMPLE 5 Finding the Midpoint

Point C is the midpoint of $\overline{AB}$.

Find AC , CB , and AB .

$$\begin{array}{ccccc}
 & 8x - 6 & & 7x + 3 & \\
 \bullet & & \bullet & & \bullet \\
 A & & C & & B
 \end{array}$$

Solution Let's use the meaning of midpoint to find the value of x .

$$\begin{array}{rcl}
 AC = CB & \text{Definition of midpoint} \\
 8x - 6 = 7x + 3 & \text{Substitute.} \\
 x - 6 = 3 & \text{Subtract } 7x \text{ from both sides.} \\
 x = 9 & \text{Add 6 to both sides.}
 \end{array}$$

Now let's use the value of x to find AC and CB , which should be the same.

$$\begin{array}{rcl}
 AC = 8x - 6 & & CB = 7x + 3 \\
 = 8(9) - 6 & \text{Substitute 9 for } x. & = 7(9) + 3 \\
 = 72 - 6 & \text{Multiply.} & = 63 + 3 \\
 = 66 & \text{Simplify.} & = 66
 \end{array}$$

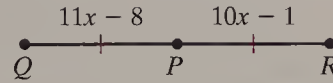
Notice that $AC = CB = 66$, as expected.

To find AB , we add.

$$\begin{array}{rcl}
 AB = AC + CB & \text{Use the Segment Addition Postulate.} \\
 = 66 + 66 & \text{Substitute.} \\
 = 132 & \text{Add.}
 \end{array}$$

PRACTICE

5

Point P is the midpoint of QR . Find QP , PR , and QR .

VOCABULARY & READINESS CHECK

Word Bank Use the choices to fill in each blank.

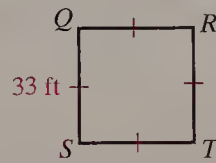
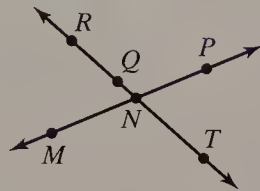
bisect distance between segment bisector
midpoint coordinate equal congruent

- Two segments that have the same length are called _____ segments.
- The real number that corresponds to a point is called the _____ of the point.
- Given three distinct points on a line, one of the points will always be _____ the other two.
- A point that divides a segment into two congruent segments is called a(n) _____.
- If two segments are congruent, then their lengths are _____.
- The _____ between two points is the absolute value of the difference of their coordinates.
- A line, ray, segment, or plane that intersects a segment at its midpoint is called a(n) _____.
- In geometry, to _____ a segment means to divide it into two equal parts.

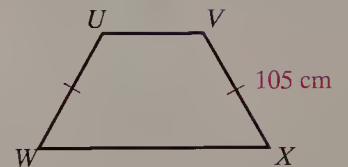
1.4 | EXERCISE SET MyMathLab®

Use the figure shown to answer each question as yes or no.

- Is N between M and P ?
- Is N between Q and T ?
- Is Q between N and T ?
- Is M between N and P ?
- Is T between M and P ?
- Is P between R and T ?
- Is R between Q and T ?
- Is N between R and Q ?
- Is Q between R and N ?
- Is Q between R and T ?

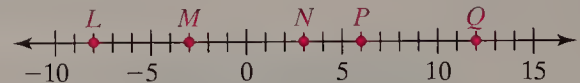


- AC
- EG
- QR



- UW
- FG
- RT

Use the number line below for Exercises 21–24. Tell whether the segments are congruent. See Example 3.



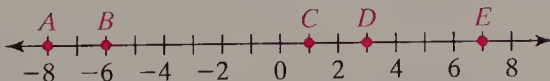
- $\overline{LN}$ and $\overline{MQ}$
- $\overline{MP}$ and $\overline{NQ}$
- $\overline{MN}$ and $\overline{PQ}$
- $\overline{LP}$ and $\overline{MQ}$

Use the number line below for Exercises 25–28. See Example 4.



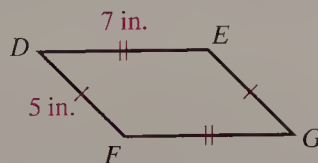
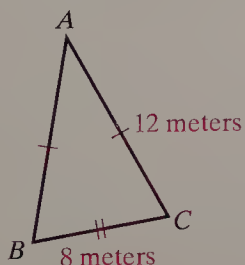
- If $RS = 15$ and $ST = 9$, then $RT =$ _____.
- If $ST = 15$ and $RT = 40$, then $RS =$ _____.
- Multiple Steps** $RS = 8y + 4$, $ST = 4y + 8$, and $RT = 96$.
 - What is the value of y ?
 - Find RS and ST .
- Multiple Steps** $RS = 14x - 8$, $ST = 9x + 10$, and $RT = 232$.
 - What is the value of x ?
 - Find RS and ST .

Find the length of each segment. See Example 1.



- $\overline{BD}$
- $\overline{AB}$
- $\overline{CE}$
- $\overline{AD}$

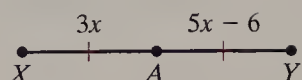
Each figure is marked separately, showing their congruent segments. Use the figures to find each measure in Exercises 15–20. See Example 2.



Multiple Steps For Exercises 29 and 30, see Example 5.

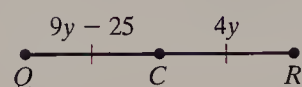
29. A is the midpoint of $\overline{XY}$.

- Find the value of x .
- Find XA , AY , and XY .



30. C is the midpoint of $\overline{QR}$.

- Find the value of y .
- Find QC , CR , and QR .



For Exercises 31–34, use the figure below. Find the value of PT . See Example 5.



- $PT = 5x + 3$ and $TQ = 7x - 9$
- $PT = 6x - 35$ and $TQ = 2x - 3$
- $PT = 4x - 6$ and $TQ = 3x + 4$
- $PT = 7x - 24$ and $TQ = 6x - 2$

On a number line, the coordinates of X , Y , Z , and W are -7 , -3 , 1 , and 5 , respectively. Find the lengths of the two segments. Then tell whether they are congruent.

- $\overline{XY}$ and $\overline{ZW}$
- $\overline{ZX}$ and $\overline{WY}$

CONCEPT EXTENSIONS

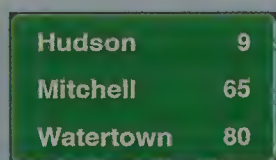
The numbers labeled on the map of Florida are mile markers. Assume that Route 10 between Quincy and Jacksonville is straight.



- Suppose you drive at an average speed of 55 mph. How long will it take to get from Live Oak to Jacksonville?
- Suppose you drive at an average speed of 58 mph. How long will it take to get from Macclenny to Tallahassee?

Find the Error Use the highway sign for Exercises 45 and 46.

- A driver reads the highway sign and says, “It’s 145 miles from Mitchell to Watertown.” What error did the driver make? Explain.



- Your friend reads the highway sign and says, “It’s 71 miles to Watertown.” Is your friend correct? Explain.
- C is the midpoint of $\overline{AB}$, D is the midpoint of $\overline{AC}$, E is the midpoint of $\overline{AD}$, F is the midpoint of $\overline{ED}$, G is the midpoint of $\overline{EF}$, and H is the midpoint of $\overline{DB}$. If $DC = 16$, what is GH ?

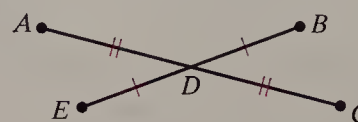
Suppose the coordinate of A is 0, $AR = 5$ units, and $AT = 7$ units. What are the possible coordinates of the midpoint of the given segment?

37. $\overline{AR}$

38. $\overline{AT}$

- Suppose point E has a coordinate of 3 and $EG = 5$ units. What are the possible coordinates of point G ?
- Suppose point P has a coordinate of 10 and $PG = 6$ units. What are the possible coordinates of point G ?

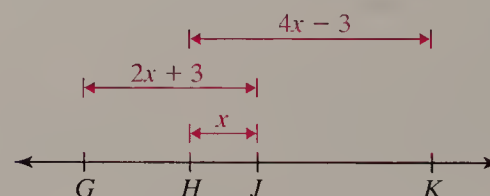
Use the diagram below for Exercises 41 and 42.



- If $AD = 12$ and $AC = 4y - 36$, find the value of y . Then find AC and DC .
- If $ED = x + 4$ and $DB = 3x - 8$, find ED , DB , and EB .

48. Multiple Steps

- Use the diagram below. What algebraic expression represents GK ?
- If $GK = 30$, what are GH and JK ?



- Suppose you know PQ and QR . Can you use the Segment Addition Postulate to find PR ? Explain.
- Use the map above for Exercises 43 and 44 and explain how to use mile markers to find distances between points.

1.5 ANGLES AND THEIR MEASURE

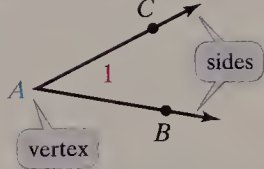
OBJECTIVES

- 1 Understand the Measure of Angles.
- 2 Use Algebra and the Angle Addition Postulate to Solve Applications and Find Angle Measures.

VOCABULARY

- angle
- sides of an angle
- vertex
- interior of an angle
- exterior of an angle
- protractor
- degrees
- acute angle
- right angle
- obtuse angle
- straight angle
- congruent angles

OBJECTIVE 1 ▶ Understanding Angle Measures. In Section 1.3 we defined rays. What can we call a figure formed by two rays that share a common endpoint? Can you see that this type of thinking leads to new figures in our geometry mathematical system? Let’s use the terms we’ve learned thus far to define new terms and figures.

Defined Terms		
Definition	How to Name It	Example
An angle consists of two different rays with a common endpoint.	 Point A is the vertex. The sides are rays $\overrightarrow{AC}$ and $\overrightarrow{AB}$.	Ways to name this angle: $\angle A$, $\angle 1$ $\angle CAB, \angle BAC$
The rays are the sides of the angle. The common endpoint is the vertex of the angle.		

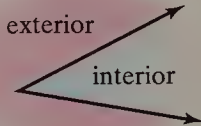
(Note: The number 1 is inserted to show a numerical way to name this angle.)

Helpful Hint

The vertex must be in the middle when naming an angle using three points.

The **interior** of an angle contains all points between the two sides of the angle.

The **exterior** of an angle contains all points that are not in the interior of the angle and are not on the angle.



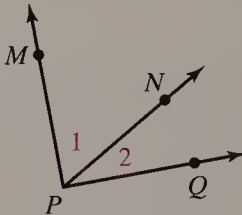
Helpful Hint

Given an angle in a plane, every point on the plane

- is in the interior of the angle,
- is in the exterior of the angle, or
- lies on the angle

EXAMPLE 1 Identifying and Naming Angles

- a. How many different angles are in the diagram?
- b. Write two other ways to name $\angle 1$.



Solution

- a. There are three different angles in the diagram: $\angle 1$, $\angle 2$, and $\angle MPQ$.
- b. $\angle 1$ can also be named as $\angle MPN$ or $\angle NPM$.

PRACTICE

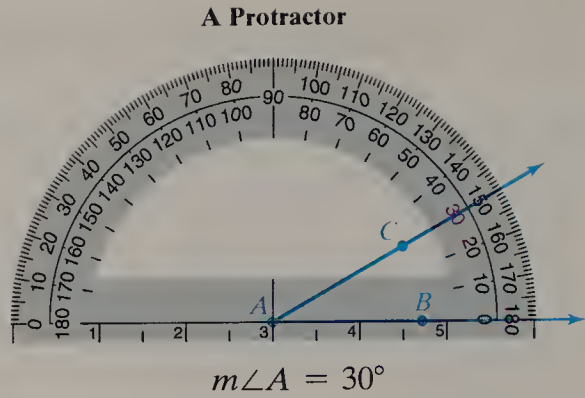
1

- a. Write two other ways to name $\angle 2$.
- b. Write one other way to name $\angle MPQ$.

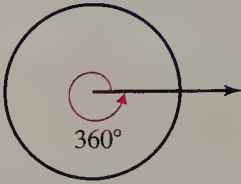
Helpful Hint

In the diagram for Example 1, no angle should be named $\angle P$ because all three angles have point P as their vertex. $\angle P$ does not clearly tell us what angle we are referring to.

The instrument to the right is called a **protractor**. It can be used to measure angles in units called **degrees** ($^\circ$). For example, the measure of $\angle A$ (also denoted by $m\angle A$) is 30° .



How do we understand the measure of one degree, or 1° ? We start with a circle. A circle has 360° ; thus 1° is $\frac{1}{360}$ of the rotation to form a circle.

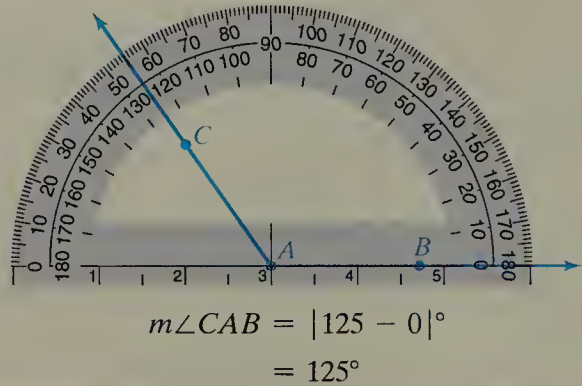


Helpful Hint

Notice that a protractor reminds us of half a circle, and shows half of 360° , or 180° .

Postulate 1.5-1 Protractor Postulate

Suppose we have $\overrightarrow{AB}$ as shown and point C on one side of $\overrightarrow{AB}$. Every ray, for example, $\overrightarrow{AC}$ can be paired one-to-one with a real number from 0 to 180. The measure of $\angle CAB$ (in degrees) equals the absolute value of the difference between the real numbers on the protractor for $\overrightarrow{AC}$ and for $\overrightarrow{AB}$.



Helpful Hint

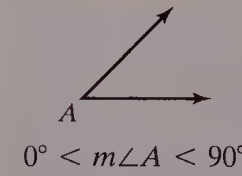
Learn how to use your protractor. Notice that there are two sets of numbering about the protractor, both from 0 to 180. There is **lower numbering** and **upper numbering**. Be consistent and use **lower numbering** for both rays or **upper numbering** for both rays.



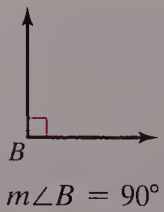
Now we can use the measure of an angle to classify it.

Classifying Angles

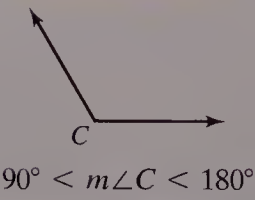
Acute angle



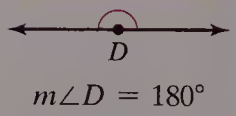
Right angle



Obtuse angle



Straight angle



Helpful Hint

The symbol $\square$ indicates a right angle.

EXAMPLE 2 Measuring and Classifying Angles

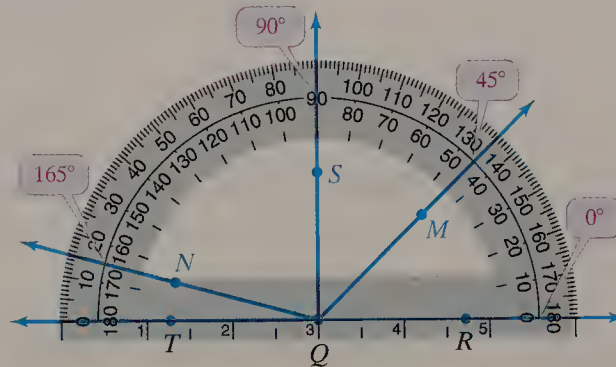
Find $m\angle RQM$, $m\angle RQS$, and $m\angle RQN$. Then classify each angle as acute, right, obtuse, or straight.

Solution

$$m\angle RQM = |45 - 0|^\circ = 45^\circ; \text{ acute angle}$$

$$m\angle RQS = |90 - 0|^\circ = 90^\circ; \text{ right angle}$$

$$m\angle RQN = |165 - 0|^\circ = 165^\circ; \text{ obtuse angle}$$



PRACTICE

2 Find $m\angle TQN$, $m\angle TQM$, and $m\angle TQR$. Then classify each angle as acute, right, obtuse, or straight.

Just as we did for segments, we can use the same notation for angles of equal (=) measure.

Two angles that have the same measure are called **congruent angles**. Recall that the symbol $\cong$ means congruent.

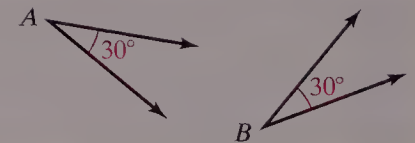
Helpful Hint

Angles are congruent, $\cong$. Angle measures are equal, $=$.

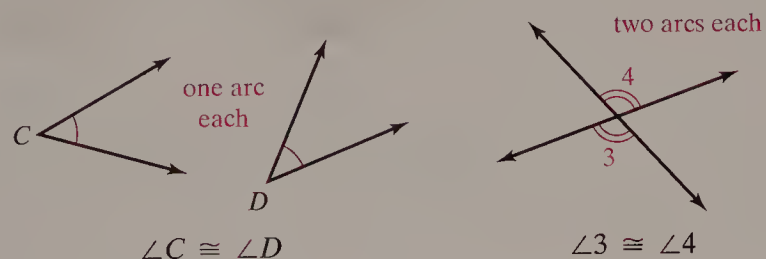
Congruent Angles vs. Equal Angle Measures

If $\angle A \cong \angle B$, then $m\angle A = m\angle B$.

Also, if $m\angle A = m\angle B$, then $\angle A \cong \angle B$.



We mark congruent angles with exactly the same number of arcs, as shown in the figure below.



Helpful Hint

There may be many angles and figures considered at one time. If so, make sure each set of congruent angles is marked with the same number of arcs.

EXAMPLE 3 Measuring Congruent Angles

Use the figures on the previous page to answer each question.

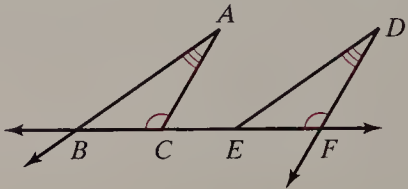
- a. If $m\angle C = 45^\circ$, find $m\angle D$.
- b. If $m\angle 3 = 113^\circ$, find $m\angle 4$.

Solution

- a. $\angle C \cong \angle D$, so $m\angle C = m\angle D$. Since $m\angle C = 45^\circ$, then $m\angle D = 45^\circ$.
- b. $\angle 3 \cong \angle 4$, so $m\angle 3 = m\angle 4$. Since $m\angle 3 = 113^\circ$, then $m\angle 4 = 113^\circ$. □

PRACTICE

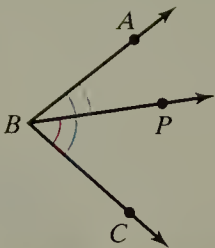
- 3**
- a. Use the figures at the right to write the congruent angles and equal angle measures.
 - b. If $m\angle A = 25^\circ$, find $m\angle D$.
 - c. If $m\angle F = 120^\circ$, find $m\angle C$.



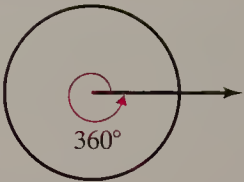
OBJECTIVE 2 ▶ Using Algebra and the Angle Addition Postulate. Notice that the Angle Addition Postulate is similar to the Segment Addition Postulate.

Postulate 1.5-2 Angle Addition Postulate

If P is in the interior of $\angle ABC$, then
 $m\angle ABP + m\angle PBC = m\angle ABC$.

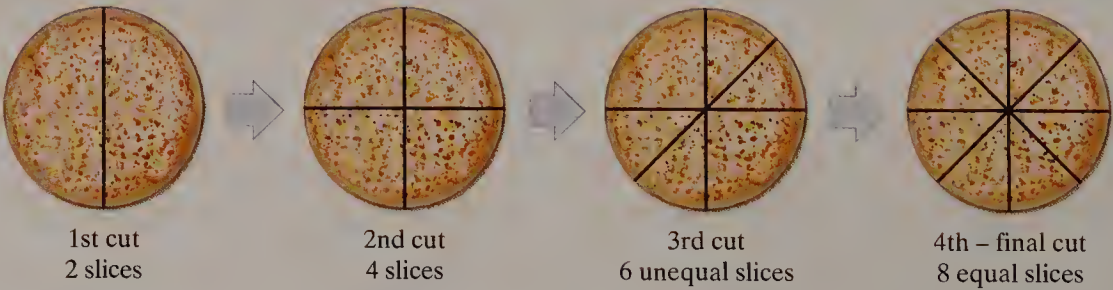


We can use our knowledge that a circle contains 360° and the Angle Addition Postulate to solve applications.



EXAMPLE 4 Dividing a Pizza Application

A circular pizza can easily be cut into 8 slices because of the ease of using a straight knife, as shown below.



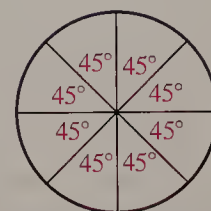
Find the angle measure of a slice of pizza cut into 8 equal-size slices, as shown.

Solution Recall that a circle measures 360° . Since each of the 8 slices has equal measure, we divide by 8.

$$\begin{aligned}\text{angle measure of a slice} &= \frac{360^\circ}{8} \\ &= 45^\circ\end{aligned}$$

If a circular pizza is cut into 8 slices, then the angle measure of each slice is 45° .

Check: To check, use the Angle Addition Postulate. Add 45° eight times, or use the definition of multiplication and see that $8 \cdot 45^\circ = 360^\circ$.



□

PRACTICE

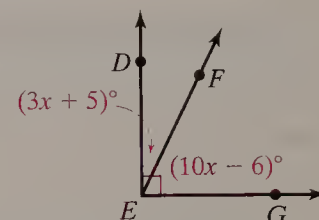
4

A circular pizza is cut into 6 equal-size slices, as shown. Find x , the angle measure of each slice.


EXAMPLE 5 Using the Angle Addition Postulate

If $\angle DEG$ is a right angle, find $m\angle DEF$ and $m\angle FEG$.

Solution Since $\angle DEG$ is a right angle, then $m\angle DEG = 90^\circ$.
Thus,



$$\begin{aligned}m\angle DEF + m\angle FEG &= m\angle DEG && \text{Angle Addition Postulate} \\ (3x + 5) + (10x - 6) &= 90 && \text{Substitution} \\ 13x - 1 &= 90 && \text{Combine like terms.} \\ 13x &= 91 && \text{Add 1 to both sides.} \\ x &= 7 && \text{Divide both sides by 13.}\end{aligned}$$

Use the value of x to find angle measures.

$$\left. \begin{aligned}m\angle DEF &= 3x + 5 = 3(7) + 5 = 26^\circ \\ m\angle FEG &= 10x - 6 = 10(7) - 6 = 64^\circ\end{aligned} \right\} \text{ Let } x = 7.$$

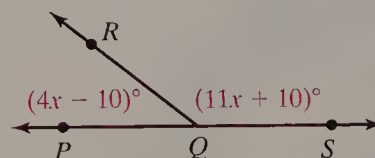
Check: $m\angle DEF + m\angle FEG = 26^\circ + 64^\circ = 90^\circ$, a right angle.

□

PRACTICE

5

If $\angle PQS$ is a straight angle, find $m\angle PQR$ and $m\angle RQS$.



VOCABULARY & READINESS CHECK

Word Bank Use the choices to fill in each blank.

vertex sides congruent
angle equal protractor

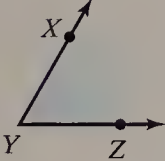
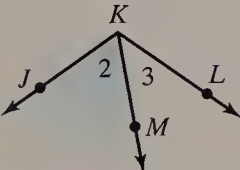
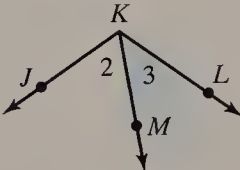
1. An instrument used to measure angles in degrees is called a(n) _____.
2. Two angles that have the same measure are called _____ angles.
3. A(n) _____ consists of two different rays with a common endpoint.
4. The rays of an angle are also called the _____ of the angle.
5. The common endpoint of the rays of an angle is called the _____ of the angle.
6. If two angles are congruent, then their measures are _____.

Matching Match each angle with the correct degree measure.

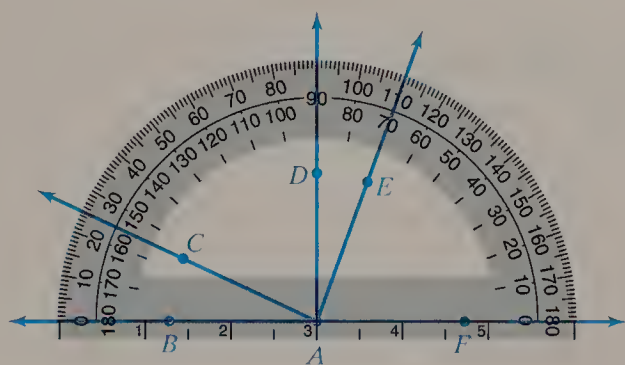
- | | |
|-------------------|---------------------------------------|
| 7. obtuse angle | A. 180° |
| 8. acute angle | B. between 90° and 180° |
| 9. straight angle | C. between 0° and 90° |
| 10. right angle | D. 90° |

1.5 EXERCISE SET MyMathLab®

Name each shaded angle in three different ways. See Example 1.

1. 
2. 
3. 
4. 

Use a Protractor Use the diagram below. Find the measure of each angle. Then classify the angle as “acute,” “right,” “obtuse,” or “straight.” See Example 2.

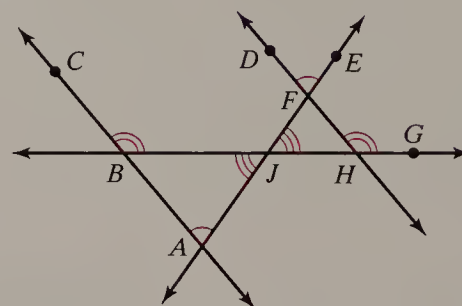


5. $\angle EAF$
6. $\angle DAF$
7. $\angle BAE$
8. $\angle BAC$
9. $\angle CAE$
10. $\angle DAE$

Sketch Draw a figure that fits each description. See Example 3.

11. an obtuse angle, $\angle RST$
12. an acute angle, $\angle GHJ$
13. a straight angle, $\angle KLM$
14. a right angle, $\angle QRS$

Use the diagram below. Use the arc marks to complete each statement. See Example 3.

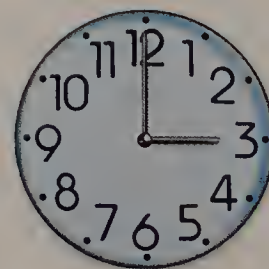


15. $\angle CBJ \cong$ _____
16. $\angle FJH \cong$ _____
17. If $m\angle EFD = 75^\circ$, then $m\angle JAB =$ _____.
18. If $m\angle GHF = 130^\circ$, then $m\angle JBC =$ _____.

Solve. See Example 4.

Use the diagram at the right to find the angle measure of the hands of a clock for each given time. (Each angle measure answer is $\leq 180^\circ$.)

19. 3:00
20. 5:00
21. 10:00
22. 11:00



Solve. See Example 4.

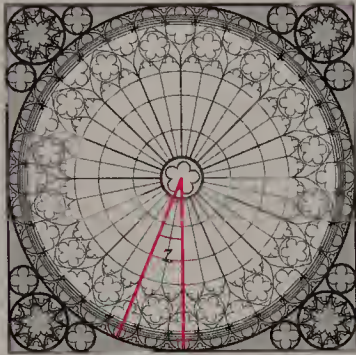
23. An orange normally has 10 segments, as shown in the cross section below. If each segment has an equal angle measure, find x , the measure of the angle of one segment.



24. An apple normally has 5 chambers (containing one or two seeds), as shown by the cross section below. If the chambers are equally placed, find the angle measure, y .



25. This rose window below is divided into 16 equal sections. Find z , the angle measure of a section.

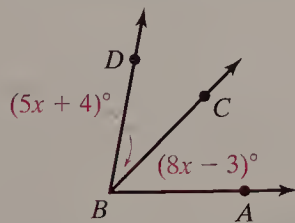


26. The wagon wheel below has 12 spokes dividing the wheel into 12 equal sections. Find x , the angle measure of a section.

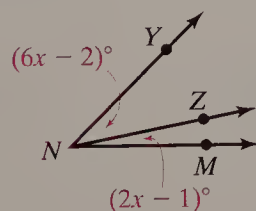


Solve Exercises 27–32. See Example 5.

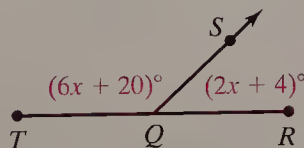
27. If $m\angle ABD = 79^\circ$, what are $m\angle ABC$ and $m\angle DBC$?



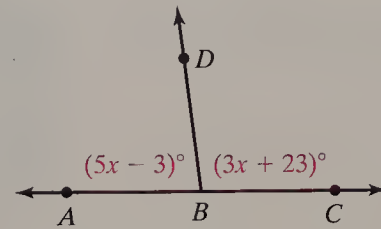
28. If $m\angle YNM = 45^\circ$, what are $m\angle YNZ$ and $m\angle ZNM$?



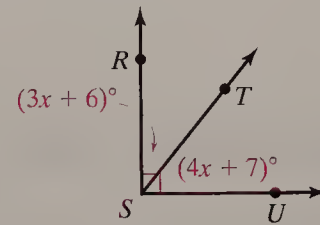
29. $\angle RQT$ is a straight angle. What are $m\angle RQS$ and $m\angle TQS$?



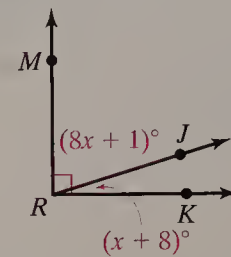
30. $\angle ABC$ is a straight angle. What are $m\angle ABD$ and $m\angle DBC$?



31. What are $m\angle RST$ and $m\angle TSU$?

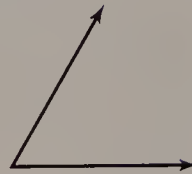


32. What are $m\angle MRJ$ and $m\angle JRK$?



Use a Protractor Measure and classify each angle. See Example 2.

33.



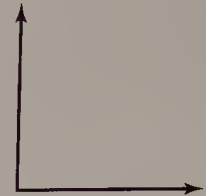
34.



35.

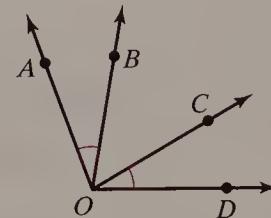


36.



CONCEPT EXTENSIONS

Use the diagram shown for Exercises 37 and 38. Solve for x . Find the angle measures to check your work. See Example 5.

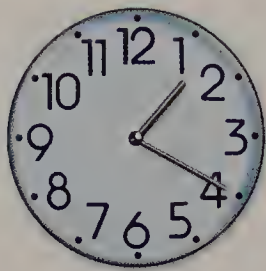


37. $m\angle AOB = (4x - 2)^\circ$, $m\angle BOC = (5x + 10)^\circ$,
 $m\angle COD = (2x + 14)^\circ$

38. $m\angle AOB = 28^\circ$, $m\angle BOC = (3x - 2)^\circ$, $m\angle AOD = 6x^\circ$

Use the diagram at the right to find the angle measure of the hands of a clock at each time. (Each angle measure is $\leq 180^\circ$.)

39. 1:20
40. 8:40
41. 3:30
42. 4:30



43. **Find the Error** Your classmate concludes from the diagram to the right that $\angle JKL \cong \angle LKM$. Is your classmate correct? Explain.



44. **Sketch** Sketch a right angle with vertex V . Name it $\angle 1$. Then sketch a 135° angle that shares a side with $\angle 1$. Name it $\angle PVB$. Is there more than one way to sketch $\angle PVB$? If so, sketch all the different possibilities. (Hint: Two angles are the same if you can rotate or flip one to match the other.)

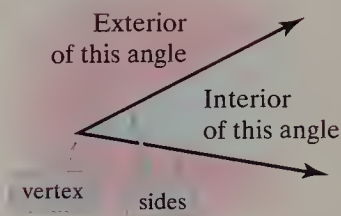
1.6 ANGLE PAIRS AND THEIR RELATIONSHIPS

OBJECTIVES

1 Learn Special Relationships Between Pairs of Angles.

2 Use Algebra to Find Angle Measures.

OBJECTIVE 1 ▶ Learning Special Relationships Between Angle Pairs. Pairs of angles have special relationships that we give special names. Before we continue, recall the different parts of an angle, shown to the right.



VOCABULARY

• adjacent angles

• vertical angles

• linear pair

• complementary angles

• complement

• supplementary angles

• supplement

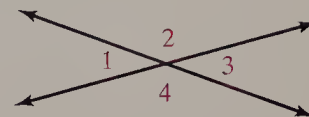
• angle bisector

Special Angle Pairs	
Definition	Examples
Two angles are called adjacent angles if they share a common side and a common vertex, but have no interior points in common.	$\angle 1$ and $\angle 2$ are adjacent angles. $\angle 3$ and $\angle 4$ are adjacent angles.
Two angles are called vertical angles if their sides form opposite rays.	$\angle 1$ and $\angle 3$ are vertical angles. $\angle 2$ and $\angle 4$ are vertical angles.
Two adjacent angles form a linear pair if their noncommon sides are opposite rays.	$\angle 5$ and $\angle 6$ form a linear pair.
Two angles are complementary if their measures have a sum of 90° . Each angle is called the complement of the other.	$\angle 1$ and $\angle 2$ are complementary angles. $\angle A$ and $\angle B$ are complementary angles.
Two angles are supplementary if their measures have a sum of 180° . Each angle is called the supplement of the other.	$\angle 3$ and $\angle 4$ are supplementary angles. $\angle C$ and $\angle D$ are supplementary angles. (Notice that these angles form a linear pair.)

EXAMPLE 1 Identifying Vertical Angles and Linear Pairs

Use the figure to answer each statement as true or false.

- a. $\angle 2$ and $\angle 3$ are vertical angles.
- b. $\angle 2$ and $\angle 4$ are vertical angles.
- c. $\angle 3$ and $\angle 4$ form a linear pair.
- d. $\angle 3$ and $\angle 1$ form a linear pair.

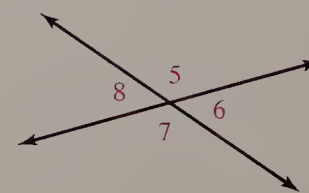
**Solution**

- a. False. The sides of the angles do not form opposite rays.
- b. True. The sides of the angles form opposite rays.
- c. True. They are adjacent angles and their noncommon sides form opposite rays.
- d. False. They are not adjacent angles.

PRACTICE**1**

Use the figure to answer each statement as true or false.

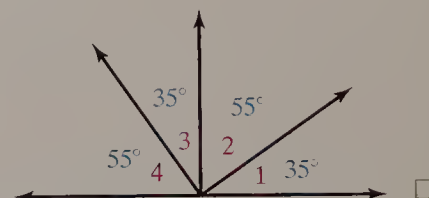
- a. $\angle 8$ and $\angle 5$ form a linear pair.
- b. $\angle 8$ and $\angle 6$ are vertical angles.
- c. $\angle 7$ and $\angle 5$ form a linear pair.
- d. $\angle 7$ and $\angle 6$ are vertical angles.

**EXAMPLE 2** Identifying Complementary Angles

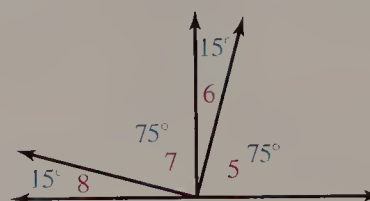
Use the figure to identify each pair of complementary angles.

Solution

- $\angle 1$ and $\angle 2$ Because $35^\circ + 55^\circ = 90^\circ$
- $\angle 2$ and $\angle 3$ Because $55^\circ + 35^\circ = 90^\circ$
- $\angle 3$ and $\angle 4$ Because $35^\circ + 55^\circ = 90^\circ$
- $\angle 1$ and $\angle 4$ Because $35^\circ + 55^\circ = 90^\circ$

**PRACTICE****2**

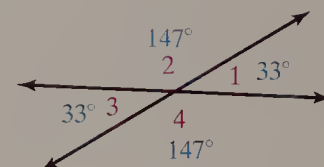
Use the figure to identify each pair of complementary angles.

**EXAMPLE 3** Identifying Supplementary Angles

Use the figure to identify each pair of supplementary angles.

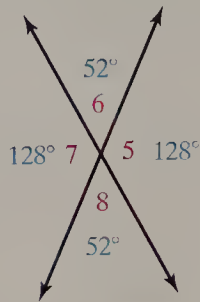
Solution

- $\angle 1$ and $\angle 2$ Because $33^\circ + 147^\circ = 180^\circ$
- $\angle 2$ and $\angle 3$ Because $147^\circ + 33^\circ = 180^\circ$
- $\angle 3$ and $\angle 4$ Because $33^\circ + 147^\circ = 180^\circ$
- $\angle 4$ and $\angle 1$ Because $147^\circ + 33^\circ = 180^\circ$



PRACTICE
3

Use the figure to identify each pair of supplementary angles.



EXAMPLE 4 Finding Measures of Complementary and Supplementary Angles

Given that $m\angle P = 73^\circ$:

- a. If $\angle A$ and $\angle P$ are supplementary angles, find $m\angle A$.
- b. If $\angle B$ and $\angle P$ are complementary angles, find $m\angle B$.

Solution

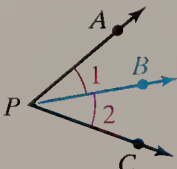
- a. Since $m\angle A + m\angle P = 180^\circ$, then $m\angle A = 180^\circ - m\angle P = 180^\circ - 73^\circ = 107^\circ$
- b. Since $m\angle B + m\angle P = 90^\circ$, then $m\angle B = 90^\circ - m\angle P = 90^\circ - 73^\circ = 17^\circ$ □

PRACTICE
4

Given that $m\angle C = 16^\circ$:

- a. If $\angle M$ and $\angle C$ are supplementary angles, find $m\angle M$.
- b. If $\angle N$ and $\angle C$ are complementary angles, find $m\angle N$.

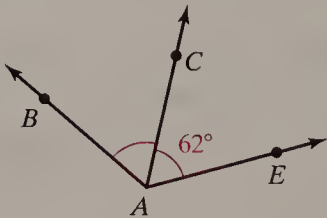
Just as a segment has a bisector (or midpoint), an angle has a bisector, called an angle bisector. An **angle bisector** is a ray that divides an angle into two adjacent angles that are congruent.

Defined Term	
Definition	Example
An angle bisector is a ray that divides an angle into two adjacent angles that are congruent.	 $\overrightarrow{PB}$ is the angle bisector of $\angle APC$. $m\angle 1 = m\angle 2$

EXAMPLE 5 Using Angle Bisectors to Find Angle Measures

Use the figure shown to find the measure of each unknown angle.

- a. Find $m\angle BAC$.
- b. Find $m\angle BAE$.



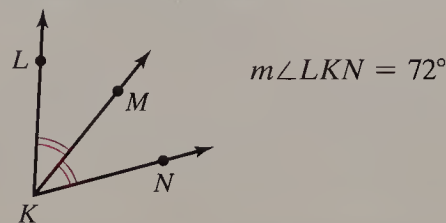
Solution From the given notation, $\angle BAC \cong \angle CAE$; thus $\overrightarrow{AC}$ is the angle bisector of $\angle BAE$. Then,

- a. $m\angle CAE = m\angle BAC = 62^\circ$.
- b. $m\angle BAE = m\angle BAC + m\angle CAE = 62^\circ + 62^\circ = 124^\circ$. □

PRACTICE

5 Use the figure shown to find the measure of each unknown angle.

- a. Find $m\angle LKM$.
b. Find $m\angle MKN$.



Helpful Hint

You may have noticed by now that vertical angles are congruent. (This is *not* a postulate, and we will prove this in Chapter 2.)

Vertical Angles:

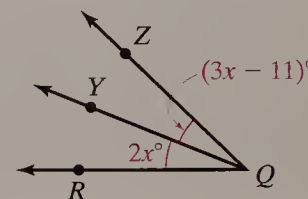


OBJECTIVE 2 Using Algebra to Find Angle Measures.

EXAMPLE 6 Finding Angle Measures

In the figure, $\overrightarrow{QY}$ bisects $\angle RQZ$.

Find the value of x ; then find $m\angle RQY$ and $m\angle YQZ$.



Solution

$$m\angle RQY = m\angle YQZ \quad \text{Congruent angles have the same measure.}$$

$$2x^\circ = (3x - 11)^\circ \quad \text{Substitution}$$

$$0 = x - 11 \quad \text{Subtract } 2x \text{ from both sides.}$$

$$11 = x \quad \text{Add 11 to both sides.}$$

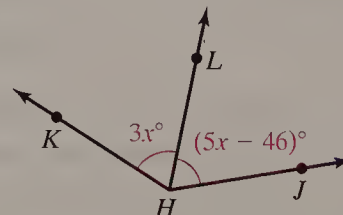
$$\left. \begin{aligned} m\angle RQY &= 2x^\circ = 2(11)^\circ = 22^\circ \\ m\angle YQZ &= (3x - 11)^\circ = (3 \cdot 11 - 11)^\circ = 22^\circ \end{aligned} \right\} \text{The values are the same, so the work is correct.} \quad \square$$

Thus, $x = 11$, and $m\angle RQY = m\angle YQZ = 22^\circ$.

PRACTICE

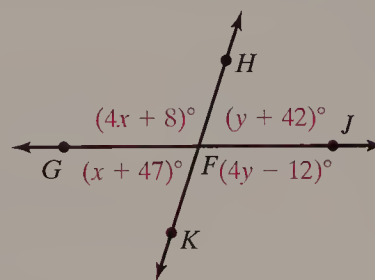
6 In the figure, $\overrightarrow{HL}$ bisects $\angle KHJ$.

Find the value of x ; then find $m\angle KHL$ and $m\angle LHJ$.



EXAMPLE 7 Finding Measures of Angles

Solve for x and y . Then find the measure of each angle.



Solution To solve for x (or y), we will choose the two angles in x (or y). These two angles form a linear pair; thus, their sum is 180° . (Angles that form a linear pair are supplementary.)

$$m\angle GFK + m\angle GFH = 180^\circ$$

$$(x + 47)^\circ + (4x + 8)^\circ = 180^\circ$$

$$5x + 55 = 180$$

$$5x = 125$$

$$x = 25$$

$$m\angle KFJ + m\angle JFH = 180^\circ$$

$$(4y - 12)^\circ + (y + 42)^\circ = 180^\circ$$

$$5y + 30 = 180$$

$$5y = 150$$

$$y = 30$$

Thus, $x = 25$ and $y = 30$.

Let's use substitution to find the measure of each angle.

$$m\angle GFK = (x + 47)^\circ = (25 + 47)^\circ = 72^\circ$$

$$m\angle GFH = (4x + 8)^\circ = (4 \cdot 25 + 8)^\circ = 108^\circ$$

$$m\angle KFJ = (4y - 12)^\circ = (4 \cdot 30 - 12)^\circ = 108^\circ$$

$$m\angle JFH = (y + 42)^\circ = (30 + 42)^\circ = 72^\circ$$

Helpful Hint

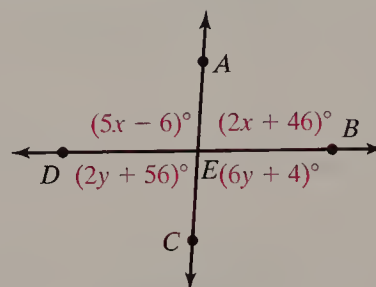
We have not formally stated nor proved anything about vertical angles. They are indeed congruent, so it is a nice way to check.

Check: Notice the angle measures of 72° , 108° , 108° , and 72° . The pairs of angles with the same measure form vertical angles, so the pairs should be congruent, which they are. Also, $72^\circ + 108^\circ = 180^\circ$ so the proper angles form linear pairs. $\square$

PRACTICE

7

Solve for x and y . Then find the measure of each angle.



VOCABULARY & READINESS CHECK

Word Bank Use the choices to fill in each blank.

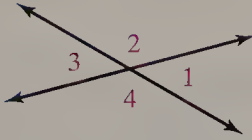
vertical angle bisector supplementary
adjacent linear pair complementary

- Two angles are _____ angles if their measures have a sum of 90° .
- Two angles are _____ angles if their measures have a sum of 180° .
- Two angles are _____ angles if their sides form opposite rays.
- Two angles are _____ angles if they share a common side and a common vertex, but have no interior points in common.
- Two adjacent angles form a(n) _____ if their noncommon sides are opposite rays.
- A ray that divides an angle into two adjacent congruent angles is called a(n) _____.

1.6 | EXERCISE SET MyMathLab®

MIXED PRACTICE

Fill in the Blank Use the diagram for Exercises 1–6. See Example 1.

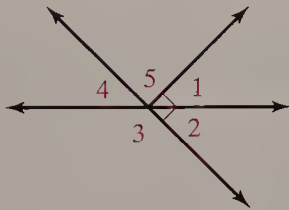


1. $\angle 2$ and _____ are vertical angles.
2. $\angle 3$ and _____ are vertical angles.

True or False Answer each statement true or false.

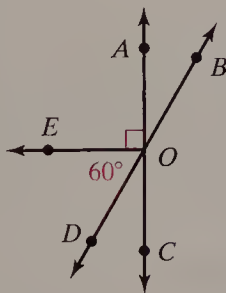
3. $\angle 4$ and $\angle 1$ form a linear pair.
4. $\angle 2$ and $\angle 1$ form a linear pair.
5. $\angle 3$ and $\angle 1$ form a linear pair.
6. $\angle 4$ and $\angle 2$ form a linear pair.

Use the diagram below. Is each statement true? Explain. See Examples 1–3.



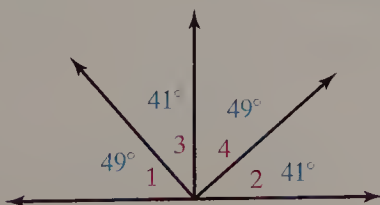
7. $\angle 1$ and $\angle 5$ are adjacent angles.
8. $\angle 3$ and $\angle 5$ are vertical angles.
9. $\angle 3$ and $\angle 4$ are complementary.
10. $\angle 1$ and $\angle 2$ are supplementary.

Name an angle or angles in the diagram described by each of the following. See Examples 1–3.



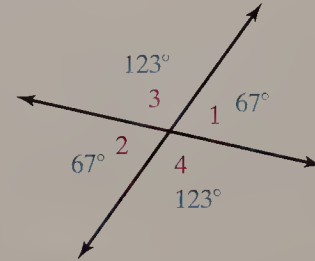
11. supplementary to $\angle AOD$
12. adjacent and congruent to $\angle AOE$
13. supplementary to $\angle EOA$
14. complementary to $\angle EOD$
15. an angle that is vertical to $\angle AOB$
16. an angle that is vertical to $\angle BOC$

True or False Use the diagram to answer each statement true or false. See Example 2.



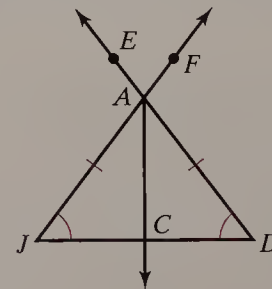
17. $\angle 2$ and $\angle 3$ are complementary angles.
18. $\angle 1$ and $\angle 4$ are complementary angles.
19. $\angle 2$ and $\angle 4$ are complementary angles.
20. $\angle 3$ and $\angle 1$ are complementary angles.
21. $\angle 1$ and $\angle 2$ are complementary angles.
22. $\angle 3$ and $\angle 4$ are complementary angles.

True or False Use the diagram to answer each statement true or false. See Example 3.



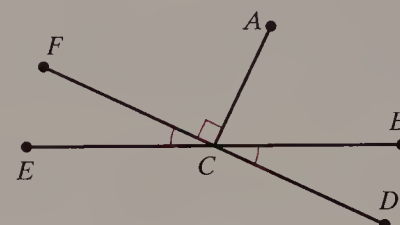
23. $\angle 1$ and $\angle 2$ are supplementary angles.
24. $\angle 3$ and $\angle 4$ are supplementary angles.
25. $\angle 3$ and $\angle 2$ are supplementary angles.
26. $\angle 4$ and $\angle 1$ are supplementary angles.

For Exercises 27–34, can you make the given conclusion from the information in the diagram? Explain. See Examples 1–3.



27. $\angle J \cong \angle D$
28. $\angle JAC \cong \angle DAC$
29. $m\angle FAD + m\angle DAJ = 180^\circ$
30. $m\angle JCA + m\angle ACD = 180^\circ$
31. $\overline{AJ} \cong \overline{AD}$
32. C is the midpoint of $\overline{JD}$.
33. $\angle JAE$ and $\angle EAF$ are vertical angles.
34. $\angle EAF$ and $\angle JAD$ are vertical angles.

In the diagram below, $m\angle ACB = 65^\circ$. Find each of the following. See Examples 1–4.



35. $m\angle ACD$
36. $m\angle BCD$
37. $m\angle ECD$
38. $m\angle ACE$

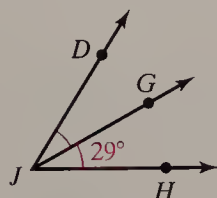
Fill in the Chart The first row has been completed for you. See Example 4.

	Angle Measure	Measure of Its Supplement	Measure of Its Complement
	30°	150°	60°
39.	10°		
40.	70°		
41.	88°		
42.	1°		
43.	123°		
44.	175°		

Use the notation on each diagram to find each angle measure. See Example 5.

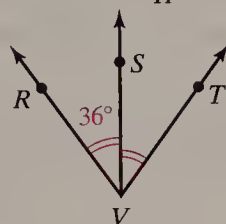
45. $m\angle DJG$

46. $m\angle DJH$



47. $m\angle RVT$

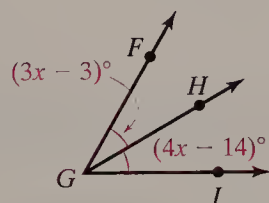
48. $m\angle SVT$



Multiple Steps Solve. See Example 6.

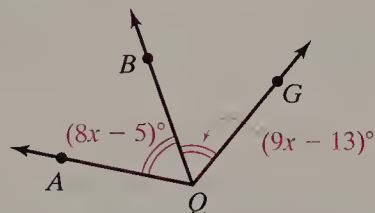
49. In the diagram, $\overrightarrow{GH}$ bisects $\angle FGI$.

- Solve for x .
- Find $m\angle FGH$.
- Find $m\angle HGI$.
- Find $m\angle FGI$.



50. In the diagram, $\overrightarrow{QB}$ bisects $\angle AQG$.

- Solve for x .
- Find $m\angle AQB$.
- Find $m\angle BQG$.
- Find $m\angle AQG$.



For Exercises 51–54, $\overrightarrow{BD}$ bisects $\angle ABC$. Solve for x and find $m\angle ABC$. See Example 6.

51. $m\angle ABD = 5x^\circ$, $m\angle DBC = (3x + 10)^\circ$

52. $m\angle ABC = (4x - 12)^\circ$, $m\angle ABD = 24^\circ$

53. $m\angle ABD = (4x - 16)^\circ$, $m\angle CBD = (2x + 6)^\circ$

54. $m\angle ABD = (3x + 20)^\circ$, $m\angle CBD = (6x - 16)^\circ$

55. **Multiple Steps** $\angle RQS$ and $\angle TQS$ are a linear pair where $m\angle RQS = (2x + 4)^\circ$ and $m\angle TQS = (6x + 20)^\circ$. See Example 6.

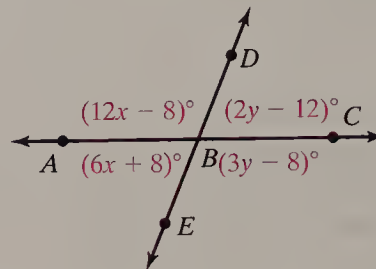
- Solve for x .
- Find $m\angle RQS$ and $m\angle TQS$.
- Show how you can check your answer.

56. **Multiple Steps** $\angle EFG$ and $\angle GFH$ are a linear pair where $m\angle EFG = (2n + 21)^\circ$, and $m\angle GFH = (4n + 15)^\circ$.

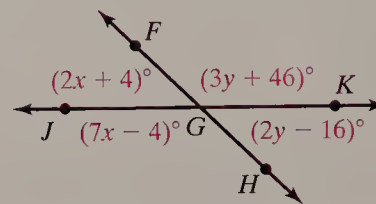
- Solve for n .
- Find $m\angle EFG$ and $m\angle GFH$.
- Show how you can check your answer.

Solve for x and y . Then find the measure of each angle. See Example 7.

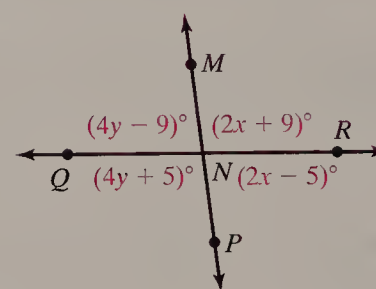
57.



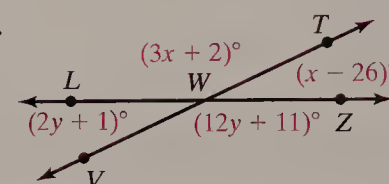
58.



59.



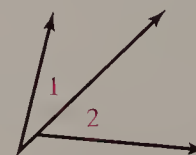
60.



CONCEPT EXTENSIONS

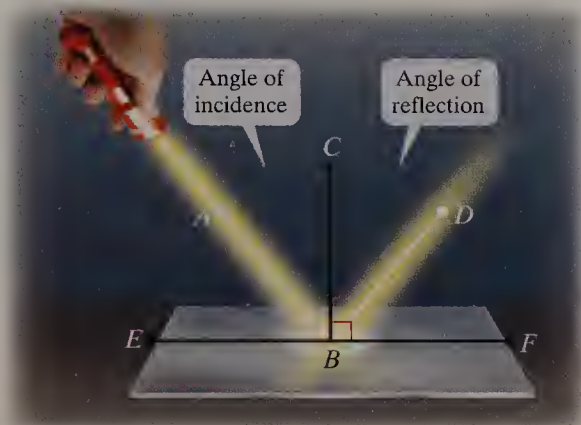
For Exercises 61 and 62, find the measure of each angle in the angle pair described. Start by drawing a diagram.

- The measure of one angle is twice the measure of its supplement.
- The measure of one angle is 20 less than the measure of its complement.
- In the diagram below, are $\angle 1$ and $\angle 2$ adjacent? Justify your reasoning.



64. When $\overrightarrow{BX}$ bisects $\angle ABC$, $\angle ABX \cong \angle CBX$. One student claims there is always a related equation $m\angle ABX = \frac{1}{2}m\angle ABC$. Another student claims the related equation is $2m\angle ABX = m\angle ABC$. Who is correct? Explain.

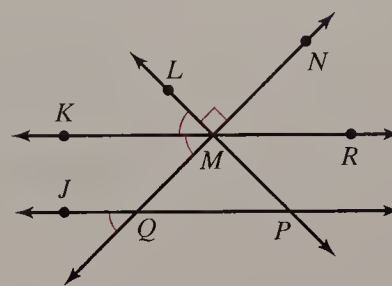
- 65. Multiple Steps** A beam of light and a mirror can be used to study the behavior of light. Light that strikes the mirror is reflected so that the angle of reflection and the angle of incidence are congruent. In the diagram, $\angle ABC$ has a measure of 41° .
- Name the angle of reflection and find its measure.
 - Find $m\angle ABD$.
 - Find $m\angle ABE$ and $m\angle DBF$.



- 66.** Describe all situations where vertical angles are also supplementary.

For Exercises 67–70, name all of the angle(s) in the diagram that are described by the following.

- 67.** adjacent and congruent to $\angle KMQ$



- 68.** supplementary to $\angle JQM$
69. complementary to $\angle NMR$
70. a linear pair with $\angle LMQ$

1.7 COORDINATE GEOMETRY—MIDPOINT AND DISTANCE FORMULAS

OBJECTIVES

- Find the Midpoint of a Segment.
- Find the Distance Between Two Points on the Coordinate Plane.

VOCABULARY

- Midpoint Formula
- Distance Formula

OBJECTIVE 1 ▶ **Finding the Midpoint of a Segment.** Recall from Section 1.4 that a **midpoint** of a segment is a point that divides, or bisects, a segment into two congruent segments.

Now we study the midpoint of a segment that is either on a number line or on the coordinate plane (also called the rectangular coordinate system).

Midpoint Formulas

	Formula	Diagram
On a Number Line The coordinate of the midpoint is the <i>average</i> or <i>mean</i> of the coordinates of the endpoints.	Given $\overline{AB}$ on a number line: The coordinate of the midpoint M of $\overline{AB}$ is $\frac{a + b}{2}$	
On the Coordinate Plane The coordinates of the midpoint are the average of the x -coordinates and the average of the y -coordinates of the endpoints.	Given $\overline{AB}$ where $A(x_1, y_1)$ and $B(x_2, y_2)$: The coordinates of the midpoint of $\overline{AB}$ are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$	

EXAMPLE 1 Finding the Midpoint on a Number Line

Use the diagram and find the coordinate of the midpoint, M , of $\overline{PQ}$.



Solution The coordinate of P is -3 . The coordinate of Q is 10 . Let $a = -3$ and $b = 10$, or simply find the average of -3 and 10 .

$$M = \frac{a + b}{2} = \frac{-3 + 10}{2} = \frac{7}{2} = 3.5$$

The coordinate, M , of the midpoint of $\overline{PQ}$ is 3.5 . □

To **check**, graph point M on the number line and see that $PM = MQ$.

PRACTICE

1 Use the diagram and find the coordinate of the midpoint, M , of $\overline{RS}$.



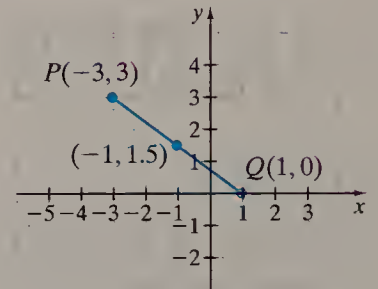
EXAMPLE 2 Finding the Midpoint in the Coordinate Plane

Find the midpoint of the line segment, $\overline{PQ}$, that joins points $P(-3, 3)$ and $Q(1, 0)$.

Solution Use the Midpoint Formula. It makes no difference which point we call (x_1, y_1) and which point we call (x_2, y_2) . Let $(x_1, y_1) = P(-3, 3)$ and $(x_2, y_2) = Q(1, 0)$.

$$\text{x-coordinate of } M = \frac{x_1 + x_2}{2} = \frac{-3 + 1}{2} = \frac{-2}{2} = -1$$

$$\text{y-coordinate of } M = \frac{y_1 + y_2}{2} = \frac{3 + 0}{2} = \frac{3}{2} = 1.5$$



The midpoint of segment $\overline{PQ}$ is $M(-1, 1.5)$, as shown on the graph. □

PRACTICE

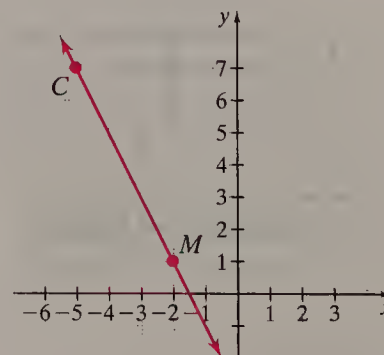
2 Find the midpoint of the line segment that joins points $P(5, -2)$ and $Q(8, -6)$.

So far, if we know the coordinates of the endpoints of a segment, we can use the **Midpoint Formula** to find the midpoint of the segment.

If we have the midpoint and an endpoint of a segment, we can also use the Midpoint Formula to find the other endpoint.

EXAMPLE 3 Finding an Endpoint

The midpoint of $\overline{CD}$ is $M(-2, 1)$. One endpoint is $C(-5, 7)$. What are the coordinates of the other endpoint, D ?



Solution Let $M(-2, 1)$ be (x, y) and $C(-5, 7)$ be (x_1, y_1) . Let the coordinates of D be (x_2, y_2) .

$$\begin{array}{rcl}
 (-2, 1) & = & \left(\frac{-5 + x_2}{2}, \frac{7 + y_2}{2} \right) \\
 \swarrow x & & \searrow y \\
 -2 & = & \frac{-5 + x_2}{2} \quad \text{Use the Midpoint Formula.} \quad 1 = \frac{7 + y_2}{2} \\
 -4 & = & -5 + x_2 \quad \text{Multiply each side by 2.} \quad 2 = 7 + y_2 \\
 1 & = & x_2 \quad \text{Solve for } x_1 \text{ and } y_2. \quad -5 = y_2
 \end{array}$$

The coordinates of D are $(1, -5)$.

To **check**, use the Midpoint Formula and see that the midpoint of segment $\overline{CD}$ is $M(-2, 1)$. $\square$

PRACTICE

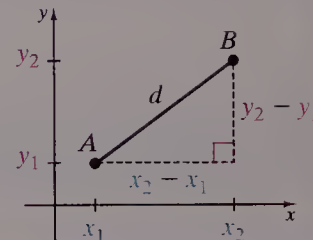
3 The midpoint of $\overline{AB}$ has coordinates $(4, -9)$. Endpoint A has coordinates $(-3, -5)$. What are the coordinates of B ?

OBJECTIVE 2 ▶ **Finding the Distance Between Two Points on the Coordinate Plane.** In Section 1.4, we learned how to find the distance between two points on a number line. To find the distance between two points on a coordinate plane, we can use the Distance Formula.

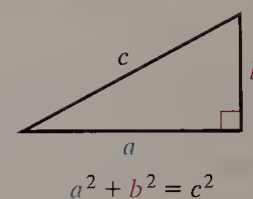
Distance Formula

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$



The **Distance Formula** is based on the *Pythagorean Theorem*, which we will study later in this book. When we use the Distance Formula, we are really finding the length of the hypotenuse of a right triangle.



EXAMPLE 4 Find the distance between $A(2, -5)$ and $B(1, -4)$. Give an exact distance and a one-decimal-place approximation.

Solution To use the Distance Formula, it makes no difference which point we call (x_1, y_1) and which point we call (x_2, y_2) . We will let $(x_1, y_1) = (2, -5)$ and $(x_2, y_2) = (1, -4)$.

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(1 - 2)^2 + [-4 - (-5)]^2} \\
 &= \sqrt{(-1)^2 + (1)^2} \\
 &= \sqrt{1 + 1} \\
 &= \sqrt{2} \approx 1.4
 \end{aligned}$$

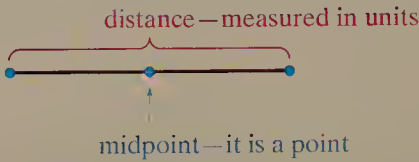
The distance between the two points is exactly $\sqrt{2}$ units, or approximately 1.4 units. $\square$

PRACTICE

4 Find the distance between $P(-3, 7)$ and $Q(-2, 3)$. Give an exact distance and a one-decimal-place approximation.

Helpful Hint

The **distance** between two points is a **distance**.
The **midpoint** of a line segment is the **point** halfway between the endpoints of the segment.



VOCABULARY & READINESS CHECK

Word Bank Use the choices below to fill in each blank. Some choices may be used more than once.

distance midpoint point

- 1. The _____ of a line segment is a _____ exactly halfway between the two endpoints of the line segment.
- 2. The _____ Formula is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
- 3. The _____ Formula for a segment on the coordinate plane is $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$.
- 4. The _____ Formula for a segment on a number line is $M = \frac{a + b}{2}$.

1.7 EXERCISE SET MyMathLab®

Find the coordinate of the midpoint of the segment with the given endpoints. See Example 1.

- 1. 2 and 4 2. -9 and 6
- 3. 2 and -5 4. -8 and -12

For Exercises 5–12, find the midpoint of the line segment whose endpoints are given. See Example 2.

- 5. $(6, -8), (2, 4)$ 6. $(3, 9), (7, 11)$
- 7. $(-2, -1), (-8, 6)$ 8. $(-3, -4), (6, -8)$
- 9. $(7, 3), (-1, -3)$ 10. $(-2, 5), (2, 15)$
- 11. $\left(\frac{1}{2}, \frac{3}{8} \right), \left(-\frac{3}{2}, \frac{5}{8} \right)$ 12. $\left(-\frac{2}{5}, \frac{7}{15} \right), \left(-\frac{2}{5}, -\frac{4}{15} \right)$

The coordinates of point T are given. The midpoint of $\overline{ST}$ is $(5, -8)$. Find the coordinates of point S . See Example 3.

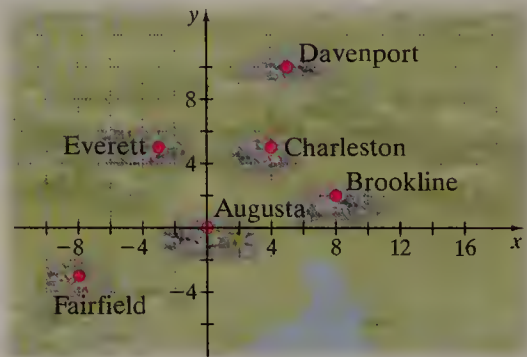
- 13. $T(0, 4)$ 14. $T(5, -15)$
- 15. $T(10, 18)$ 16. $T(-2, 8)$

Find the distance between each pair of points. For Exercises 23 and 24, give an exact answer and a one-decimal-place approximation. See Example 4.

- 17. $J(2, -1), K(2, 5)$ 18. $L(10, 14), M(-8, 14)$
- 19. $N(-1, -11), P(-1, -3)$ 20. $A(0, 3), B(0, 12)$

- 21. $E(6, -2), F(-2, 4)$ 22. $Q(12, -12), T(5, 12)$
- 23. $R(0, 5), S(12, 3)$ 24. $C(12, 6), D(-8, 18)$

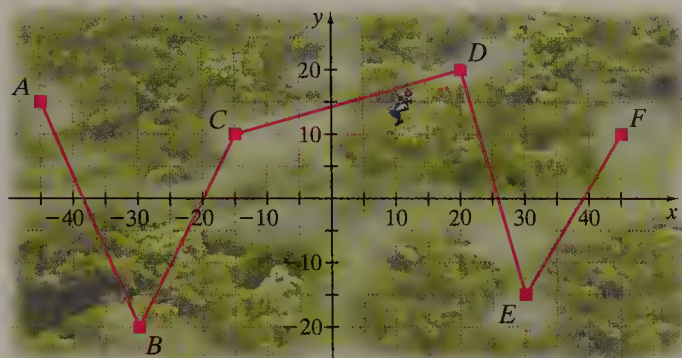
For Exercises 25–28, use the map shown. Find the distance between the cities to the nearest tenth.



- 25. Augusta and Brookline
- 26. Brookline and Charleston
- 27. Brookline and Davenport
- 28. Everett and Fairfield

On a zip-line course, you are harnessed to a cable that travels through the treetops. You start at platform A and zip to each of the other platforms. Each grid unit represents 5 m. Round each answer to the nearest tenth.

29. How far do you travel from Platform B to Platform C?



30. How far do you travel from Platform D to Platform E?

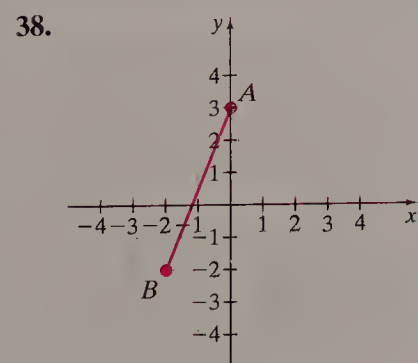
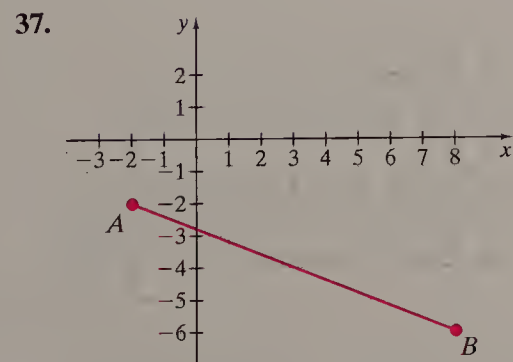
MIXED PRACTICE

Multiple Steps For Exercises 31–36, find (a) PQ to the nearest tenth and (b) the coordinates of the midpoint of $\overline{PQ}$.

31. $P(3, 2)$, $Q(6, 6)$
32. $P(-3, -1)$, $Q(5, -7)$
33. $P(0, -2)$, $Q(3, 3)$
34. $P(-4, -2)$, $Q(1, 3)$
35. $P(2, 3)$, $Q(4, -2)$
36. $P(-5, -3)$, $Q(-3, -5)$

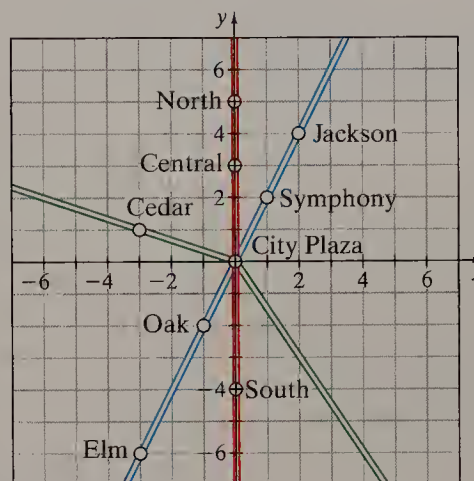
MIXED PRACTICE

Multiple Steps For each graph, find (a) AB to the nearest tenth and (b) the coordinates of the midpoint of $\overline{AB}$.



The units of the subway map below are in miles. Suppose the routes between stations are straight. Find the distance you would travel between each pair of stations to the nearest tenth of a mile.

39. Oak Station and Jackson Station



40. Central Station and South Station

41. Elm Station and Symphony Station

42. Cedar Station and City Plaza Station

43. Maple Station (not shown) is located 6 mi west and 2 mi north of City Plaza. What is the distance between Cedar Station and Maple Station?

44. Sycamore Station (not shown) is located 4 mi east and 6 mi south of City Plaza. What is the distance between Cedar Station and Sycamore Station?

CONCEPT EXTENSIONS

45. **Multiple Steps** Point $H(2, 2)$ is the midpoint of many segments.

a. Find the coordinates of the endpoints of four noncollinear segments that have point H as their midpoint.

- b. You know that a segment with midpoint H has length 8. How many possible noncollinear segments match this description? Explain.

46. **Multiple Steps** Points $P(-4, 6)$, $Q(2, 4)$, and R are collinear. One of the points is the midpoint of the segment formed by the other two points.

a. What are the possible coordinates of R ?

- b. $RQ = \sqrt{160}$. Does this information affect your answer to part a? Explain.

47. The midpoint of $\overline{TS}$ is the origin. Point T is located in quadrant II. What quadrant contains point S ? Explain.

48. How does the Distance Formula ensure that the distance between two different points is positive?

49. Do you use the Midpoint Formula or the Distance Formula to find the following?

a. Given points K and P , find the distance from K to the midpoint of $\overline{KP}$.

b. Given point K and the midpoint of $\overline{KP}$, find KP .

50. **Find the Error** Your friend calculates the distance between points $Q(1, 5)$ and $R(3, 8)$. What is his error?

$$\begin{aligned} d &= \sqrt{(1-8)^2 + (5-3)^2} \\ &= \sqrt{(-7)^2 + 2^2} \\ &= \sqrt{49 + 4} \\ &= \sqrt{53} \approx 7.3 \end{aligned}$$

1.8 CONSTRUCTIONS—BASIC GEOMETRY CONSTRUCTIONS

OBJECTIVE

- 1 Make Basic Constructions Using a Straight Edge and a Compass.

VOCABULARY

- straight edge
- compass
- construction
- perpendicular lines
- perpendicular bisector

Helpful Hint

A ruler makes a great straight edge—just don't use the measurement markings on it.

OBJECTIVE 1 ► Make Basic Constructions Using a Straight Edge and a Compass.

We can use special geometric tools to make a figure that is congruent to an original figure without measuring. This method is more accurate than sketching, drawing, or measuring.

A **straight edge** is a ruler with no markings on it. A **compass** is a geometric tool used to draw circles and parts of circles called *arcs*. A **construction** is a geometric figure drawn using a straight edge and a compass.

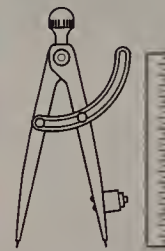
A compass is also used as a symbol of precision. Look carefully and you can see it in many icons, such as a mason symbol and many computer icons, as shown below.



Mason Symbol



East German flag



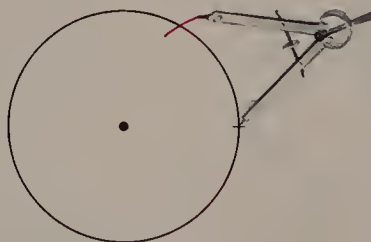
It is very important to note that we use compass and straight edge constructions to illustrate the theory of constructions. In theory, the ideal compass can draw a perfect circle, but in reality, such an instrument does not exist. To see this for yourself, use your compass and try to draw a perfect circle.

To practice using your compass, you may want to try the activity below.

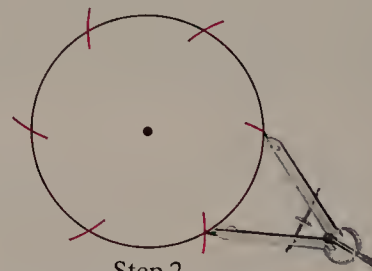
TRY THE ACTIVITY BELOW

Activity—A Compass Design

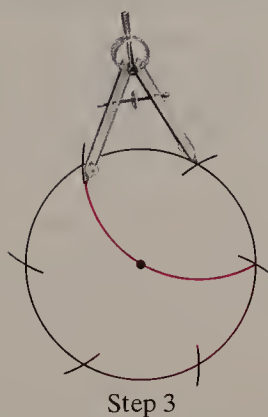
- STEP 1.** Open your compass to about 2 in. Make a circle and mark the point at the center of the circle. Keep the opening of your compass fixed. Place the compass point on the circle. With the pencil end, make a small arc to intersect the circle.
- STEP 2.** Place the compass point on the circle at the arc. Mark another arc. Continue around the circle this way to draw four more arcs—six in all.
- STEP 3.** Place your compass point on an arc you marked on the circle. Place the pencil end at the next arc. Draw a large arc that passes through the circle's center and continues to another point on the circle. (See illustration on the next page.)
- STEP 4.** Draw six large arcs in this manner, each centered at one of the six points marked on the circle. The end result is shown in color.



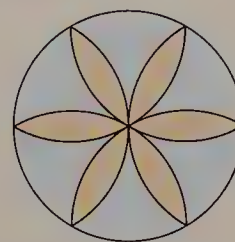
Step 1



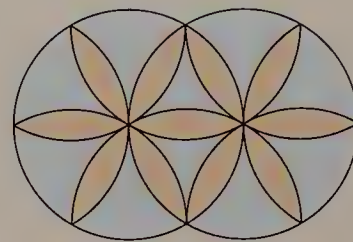
Step 2



Step 3



Step 4



Suggestion: You may extend your design by using one of the six points on the circle as the center for a new circle. Repeat Steps 1–4 with this circle. Repeat several times to make interlocking circles.

EXAMPLE 1 Constructing Congruent Segments

Construct a segment congruent to a given segment.

Given: $\overline{AB}$

Construct: $\overline{CD}$ so that $\overline{CD} \cong \overline{AB}$



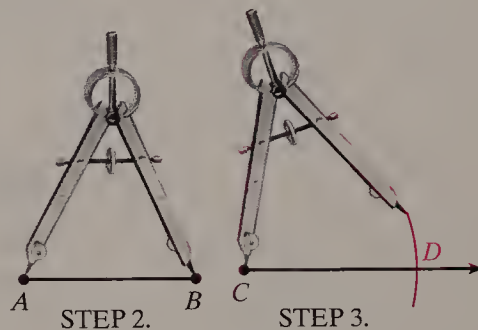
Solution

STEP 1. Draw a ray with endpoint C .



STEP 2. Open the compass to the length of $\overline{AB}$.

STEP 3. With the same compass setting, put the compass point on point C . Draw an arc that intersects the ray. Label the point of intersection D .



The segments are congruent, or $\overline{CD} \cong \overline{AB}$.

PRACTICE

1 Given $\overline{XY}$, construct $\overline{RS}$ so that $RS = 2XY$.

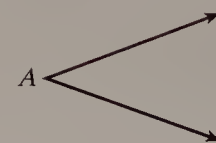


EXAMPLE 2 Constructing Congruent Angles

Construct an angle congruent to a given angle.

Given: $\angle A$

Construct: $\angle S$ so that $\angle S \cong \angle A$



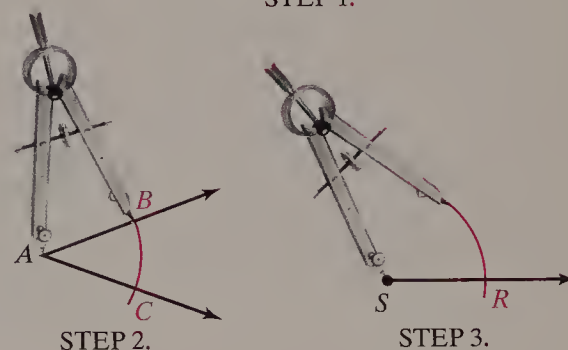
Solution

STEP 1. Draw a ray with endpoint S .

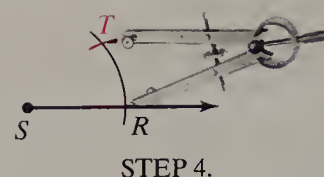


STEP 2. With the compass point on vertex A , draw an arc that intersects the sides of $\angle A$. Label the points of intersection B and C .

STEP 3. With the same compass setting, put the compass point on point S . Draw an arc and label its point of intersection with the ray as R .

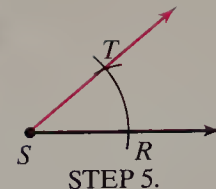


STEP 4. Open the compass to the length BC . Keeping the same compass setting, put the compass point on R . Draw an arc to locate point T .



STEP 4.

STEP 5. Draw $\overrightarrow{ST}$.

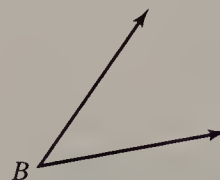


STEP 5.

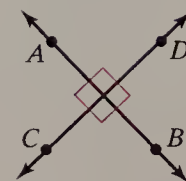
The angles are congruent, or $\angle S \cong \angle A$. □

PRACTICE

2 Given $\angle B$, construct $\angle Q$ so that $\angle Q \cong \angle B$.



Perpendicular lines are two lines that intersect to form right angles. The symbol $\perp$ means “is perpendicular to.” In the diagram at the right, $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ and $\overleftrightarrow{CD} \perp \overleftrightarrow{AB}$.



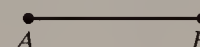
A **perpendicular bisector** of a segment is a line, segment, ray, or even a plane that is perpendicular to the segment at its midpoint. In the diagram at the right, $\overleftrightarrow{EF}$ is the perpendicular bisector of $\overline{GH}$. The perpendicular bisector bisects the segment into two congruent segments. This construction is demonstrated in Example 3. We will justify the steps for this construction in Chapter 5, as well as for the other constructions in this section.

**EXAMPLE 3** Constructing the Perpendicular Bisector

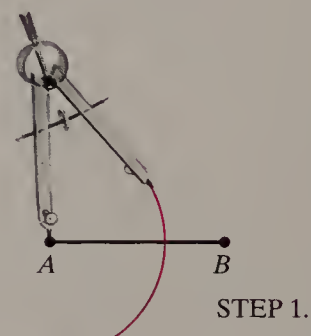
Construct the perpendicular bisector of a segment.

Given: $\overline{AB}$

Construct: $\overleftrightarrow{XY}$ so that $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$.

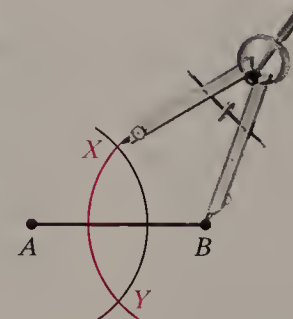
**Solution**

STEP 1. Put the compass point on point A and draw a long arc as shown. Be sure the opening is greater than $\frac{1}{2}AB$.



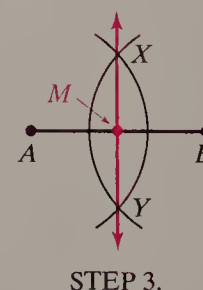
STEP 1.

STEP 2. With the same compass setting, put the compass point on point B and draw another long arc. Label the points where the two arcs intersect as X and Y .



STEP 2.

STEP 3. Draw $\overleftrightarrow{XY}$. Label the point of intersection of $\overline{AB}$ and $\overleftrightarrow{XY}$ as M , the midpoint of $\overline{AB}$.



STEP 3.

$\overleftrightarrow{XY} \perp \overline{AB}$ at midpoint M , so $\overleftrightarrow{XY}$ is the perpendicular bisector of $\overline{AB}$. □

PRACTICE

3 Draw $\overline{ST}$. Construct its perpendicular bisector.

Helpful Hint

Why must the compass opening be greater than $\frac{1}{2}AB$? If the opening is less than $\frac{1}{2}AB$, the two arcs will not intersect in Step 2.

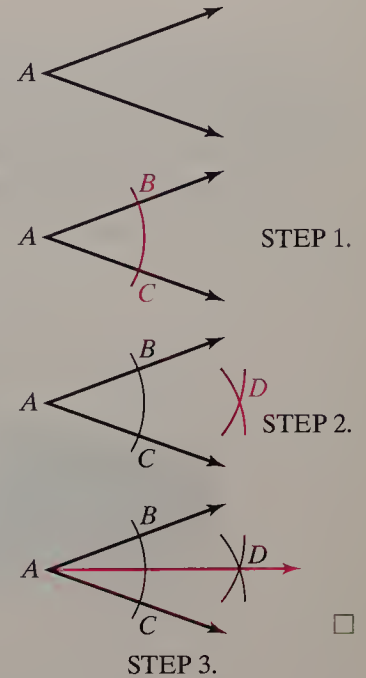
Recall that in Section 1.6, we first introduced the angle bisector.

EXAMPLE 4 Constructing the Angle Bisector

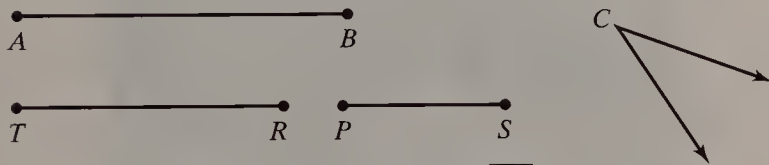
Construct the bisector of an angle.

Given: $\angle A$
Construct: $\overrightarrow{AD}$, the bisector of $\angle A$
Solution
STEP 1. Put the compass point on vertex A . Draw an arc that intersects the sides of $\angle A$. Label the points of intersection B and C .

STEP 2. Put the compass point on point C and draw an arc. With the same compass setting, draw an arc using point B . Be sure the arcs intersect. Label the point where the two arcs intersect as D .

STEP 3. Draw $\overrightarrow{AD}$.
 $\overrightarrow{AD}$ is the angle bisector of $\angle CAB$.

PRACTICE
4 Draw obtuse $\angle XYZ$. Then construct its bisector $\overrightarrow{YP}$.

1.8 | EXERCISE SET MyMathLab®

MIXED PRACTICE
Construction When needed, for Exercises 1–8, draw diagrams similar to the given ones. Then do the construction. Check your work with a ruler or a protractor. See Examples 1–4.


- Construct $\overline{XY}$ congruent to given $\overline{AB}$.
- Construct $\overline{VW}$ so that $VW = 2AB$.
- Construct $\overline{DE}$ so that $DE = TR + PS$.
- Construct $\overline{QJ}$ so that $QJ = TR - PS$.
- Construct $\angle D$ so that $\angle D \cong \angle C$.
- Construct $\angle F$ so that $m\angle F = 2m\angle C$.
- Construct the perpendicular bisector of $\overline{AB}$.
- Construct the perpendicular bisector of $\overline{TR}$.
- Draw acute $\angle PQR$. Then construct its bisector.
- Draw obtuse $\angle XQZ$. Then construct its bisector.

Construction Sketch the figure described. Explain how to construct it. Then do the construction. See Examples 1–4.

- $\overleftrightarrow{XY} \perp \overleftrightarrow{YZ}$
- $\overleftrightarrow{ST}$ bisects right $\angle PSQ$.

CONCEPT EXTENSIONS

- How is constructing an angle bisector similar to constructing a perpendicular bisector?
- Given an angle, $\angle A$, how can you construct an angle whose measure is $\frac{1}{4}m\angle A$?

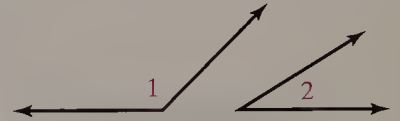
Answer the questions about a segment in a plane. Explain each answer.

- How many midpoints does the segment have?

- How many bisectors does it have?
- How many lines in the plane are its perpendicular bisectors?
- How many lines in space are its perpendicular bisectors?

Construction For Exercises 19–20, copy $\angle 1$ and $\angle 2$. Construct each angle described.

 19. $\angle B$; $m\angle B = m\angle 1 + m\angle 2$

 20. $\angle C$; $m\angle C = m\angle 1 - m\angle 2$


- Given $\overline{PQ}$, explain how to divide $\overline{PQ}$ into four congruent segments with a compass and straight edge.
- Draw a large triangle with three acute angles. Construct the bisectors of the three angles. What appears to be true about the three angle bisectors?

Construction Use a ruler to draw segments of 2 cm, 4 cm, and 5 cm. Then construct each triangle with the given side measures, if possible. If it is not possible, explain why not.

- 4 cm, 4 cm, and 5 cm
- 2 cm, 5 cm, and 5 cm
- 2 cm, 2 cm, and 5 cm
- 2 cm, 2 cm, and 4 cm
- 5 cm, 5 cm, and 5 cm
- 4 cm, 4 cm, and 4 cm

- Describe how to construct a 45° angle using what you know.
- Which step best describes how to construct the pattern at the right?

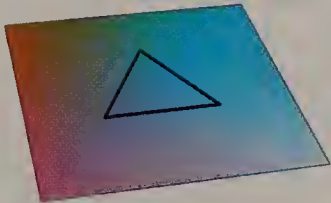
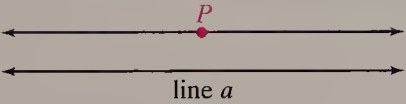
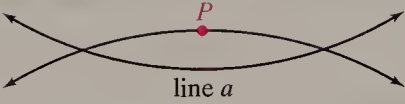
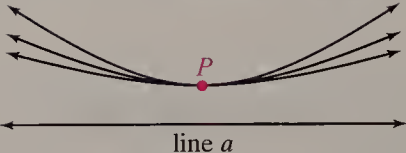


- Use a straight edge to draw the segment and then a compass to draw five half circles.
- Use a straight edge to draw the segment and then a compass to draw six half circles.
- Use a compass to draw five half circles and then a straight edge to join their ends.
- Use a compass to draw six half circles and then a straight edge to join their ends.

Historical Note—The Current Status of Geometry

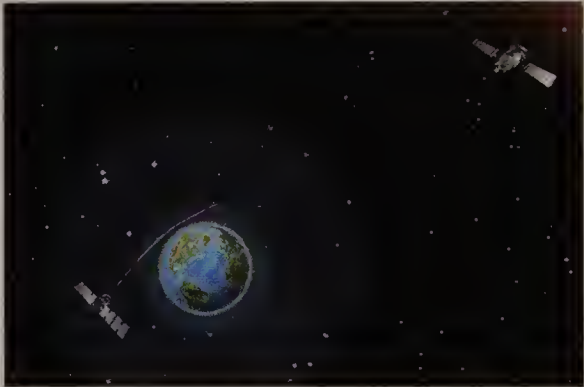
In about 1830, mathematicians János Bolyai (Hungarian, 1802–1860) and Nikolai Lobachevsky (Russian, 1792–1856) separately published work on non-Euclidean geometry. Their work is based on Euclid’s Fifth Postulate (also called the Parallel Postulate) not being valid. (See Chapter 3 of this text.)

This led, along with mathematician Georg Riemann (German 1826–1866), to two popular Non-Euclidean Geometries summarized below.

Non-Euclidean Geometries		
Euclidean Geometry	Spherical or Elliptic Geometry	Hyperbolic Geometry
 Triangle Angle Sum is 180°	 Triangle Angle Sum is greater than 180°	 Triangle Angle Sum is less than 180°
 1 parallel line through point P to line a.	 No parallel lines through point P to line a.	 Infinite number of parallel lines through point P to line a.



Hmmm...
What is the geometry
of our universe?



In 1919, the slight bending of a line was verified by the bending of starlight during a solar eclipse. This knowledge is now incorporated in appropriate applications such as the software for GPS systems.



Unfortunately, we don’t know whether light rays are proper models of Euclid’s geometry. Also, at this time, there is no test to do so.

Thus, the question still remains....

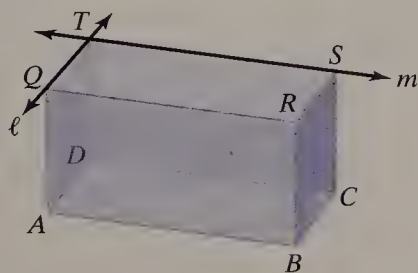
What is the geometry of our universe—Euclidean or some non-Euclidean?

CHAPTER 1 REVIEW

(1.2) Look for a pattern. If a figure pattern, sketch the next figure. If a number pattern, predict the next two numbers.

1.  , ...
2.  , ...
3. $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}, \frac{1}{24}, \dots$
4. 40, 35, 30, 25, ...

(1.3) Use the figure below for Exercises 5–8.

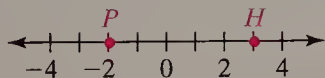


5. Name two intersecting lines.
6. Name the intersection of planes $QRBA$ and $TSRQ$.
7. Name three noncollinear points.
8. Name the intersection of lines l and m .

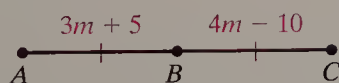
Determine whether the statement is true or false. Explain your reasoning.

9. Two points are always collinear.
10. $\overrightarrow{LM}$ and $\overrightarrow{ML}$ are the same ray.

(1.4) For Exercises 11 and 12, use the number line below.



11. Find two possible coordinates of Q such that $PQ = 5$.
12. Find the coordinate of the midpoint of $\overline{PH}$.
13. Find the value of m .



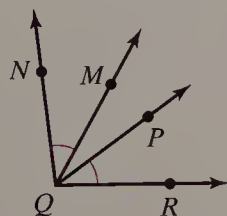
14. If $XZ = 50$, what are XY and YZ ?



(1.5) Classify each angle as “acute,” “right,” “obtuse,” or “straight.”

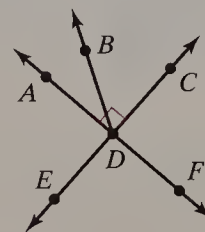
15. 
16. 

Use the diagram below for Exercises 17 and 18.



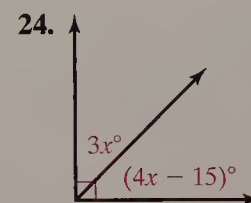
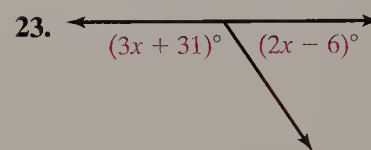
17. If $m\angle MQR = 61^\circ$ and $m\angle MQP = 25^\circ$, find $m\angle PQR$.
18. If $m\angle NQM = (2x + 8)^\circ$ and $m\angle PQR = (x + 22)^\circ$, find the value of x .

(1.6) Name a pair of each of the following.



19. complementary angles
20. supplementary angles
21. vertical angles
22. linear pair

Find the value of x .



(1.7) Find the distance between the points to the nearest tenth.

25. $A(-1, 5), B(0, 4)$
26. $C(-1, -1), D(6, 2)$

$\overline{AB}$ has endpoints $A(-3, 2)$ and $B(3, -2)$.

27. Find the coordinates of the midpoint of $\overline{AB}$.
28. Find AB to the nearest tenth.

M is the midpoint of $\overline{JK}$. Find the coordinates of K .

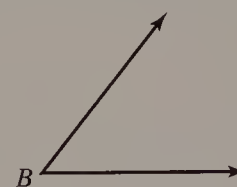
29. $J(-8, 4), M(-1, 1)$
30. $J(9, -5), M(5, -2)$

(1.8)

31. Use a protractor to draw a 73° angle. Then construct an angle congruent to it.
32. Use a protractor to draw a 60° angle. Then construct the bisector of the angle.
33. Sketch $\overline{LM}$ on paper. Construct a line segment congruent to $\overline{LM}$. Then construct the perpendicular bisector of your line segment.

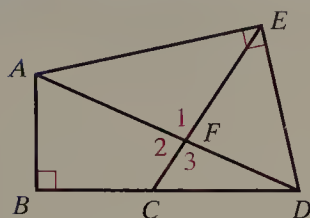


34. a. Sketch $\angle B$ on paper. Construct an angle congruent to $\angle B$.
b. Construct the bisector of your angle from part a.



d. 168°

Fill in the Blank For Exercises 8–10, use the figure below.



8. $\angle EFD$ and _____ are vertical angles.
9. $\angle EFD$ and _____ form a linear pair.
10. The complementary angles are _____

True or False Answer Exercises 11–15 true or false.

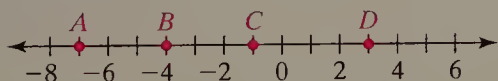
11. Angles that form a linear pair are supplementary.
12. Complementary angles are always congruent.
13. Vertical angles are always congruent.
14. Right angles are always congruent.
15. Obtuse angles are always congruent.

Label each angle below as “acute,” “right,” “obtuse,” or “straight.”

- 16.
- 17.
- 18.
- 19.

Use the number line for Exercises 20–21.

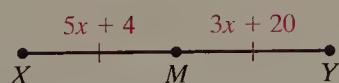
20. Find the length of $\overline{AD}$.



21. Decide whether segments $\overline{AC}$ and $\overline{BD}$ are congruent.
22. M is the midpoint of $\overline{XY}$.

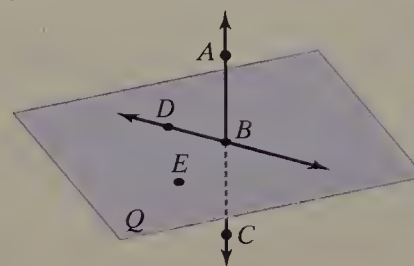
a. Find the value of x .

b. Find XM , MY , and XY .

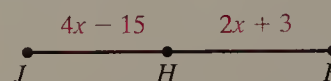


23. **Construction** Draw an obtuse $\angle ABC$. Use a compass and a straight edge to bisect the angle.

Use the figure for Exercises 24–27.

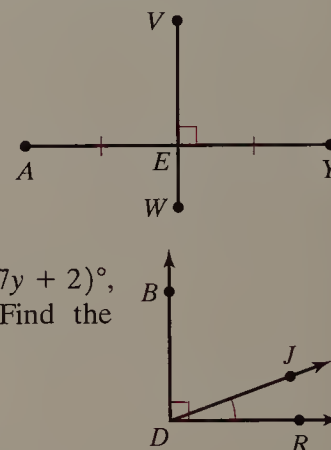


24. Name three collinear points.
25. What is the intersection of $\overleftrightarrow{AC}$ and plane Q ?
26. How many planes contain the given line and point?
 - a. $\overleftrightarrow{DB}$ and point A
 - b. $\overleftrightarrow{AC}$ and point D
27. Name four coplanar points.
28. If $JK = 48$, find the value of x .



Fill in the Blank Use the figure to complete each statement.

29. $\overline{VW}$ is the _____ of $\overline{AY}$.
30. If $EY = 3.5$, then $AY =$ _____.
31. _____ is the midpoint of _____.
32. In the diagram, $m\angle BDJ = (7y + 2)^\circ$, and $m\angle JDR = (2y + 7)^\circ$. Find the value of y .
33. $\overrightarrow{PB}$ bisects $\angle RPT$ so that $m\angle RPB = (x + 2)^\circ$ and $m\angle TPB = (2x - 6)^\circ$. Draw a figure; then find $m\angle RPT$.
34. $\overline{RS}$ has endpoints $R(2, 4)$ and $S(-1, 7)$. What are the coordinates of its midpoint, M ?
35. The midpoint of $\overline{BC}$ is $(5, -2)$. One endpoint is $B(3, 4)$. What are the coordinates of endpoint C ?
36. What is the distance between points $K(-9, 8)$ and $L(-6, 0)$?



CHAPTER 1 STANDARDIZED TEST

Look for a pattern and predict the next two numbers.

1. 4, 5, 7, 10, ...

a. 14, 19

b. 11, 13

c. 14, 18

d. 15, 21

2. $\frac{1}{2}, \frac{1}{6}, \frac{1}{18}, \frac{1}{54}, \dots$

a. $\frac{1}{162}, \frac{1}{486}$

b. $\frac{1}{108}, \frac{1}{216}$

c. $\frac{1}{108}, \frac{1}{324}$

d. $\frac{1}{162}, \frac{1}{324}$

Choose the best word or phrase to complete each statement.

3. A word that is described but not defined is a(n) _____.

a. theorem

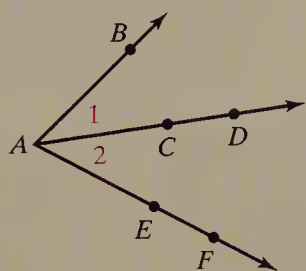
b. axiom

c. undefined term

d. defined term

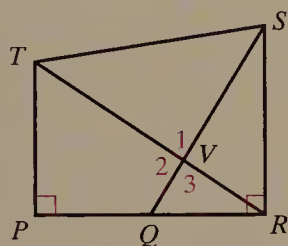
4. The number of noncollinear points needed to describe a plane is _____.
- a. one b. two
c. three d. four

Use this figure for Questions 5–7.



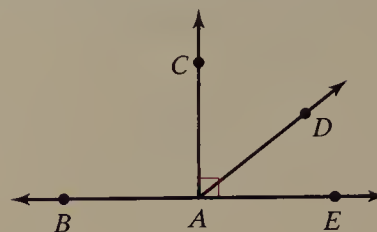
5. Another way to describe $\angle BAC$ is _____.
- a. $\angle BAE$ b. $\angle BAD$
c. $\angle 2$ d. $\angle BAF$
6. _____ is between A and D.
- a. $\overrightarrow{AC}$ b. point C
c. point B d. $\overrightarrow{AF}$
7. $\overrightarrow{AD}$ bisects $\angle BAE$. If $m\angle 1 = (3x - 2)^\circ$ and $m\angle 2 = (4x - 7)^\circ$, then $m\angle BAE =$ _____.
- a. 5° b. 13°
c. 30° d. 26°

Use the figure for Questions 8–11.



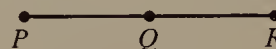
8. A pair of vertical angles is _____.
- a. $\angle 1$ and $\angle 2$ b. $\angle SRQ$
c. $\angle SRV$ and $\angle VRQ$ d. $\angle 1$ and $\angle 3$
9. A pair of complementary angles is _____.
- a. $\angle 1$ and $\angle 2$ b. $\angle SRQ$ and $\angle TPQ$
c. $\angle SRV$ and $\angle VRQ$ d. $\angle 1$ and $\angle 3$
10. _____ form a linear pair.
- a. $\angle 1$ and $\angle 2$ b. $\angle SRQ$ and $\angle TPQ$
c. $\angle SRV$ and $\angle VRQ$ d. $\angle 1$ and $\angle 3$
11. _____ are congruent angles.
- a. $\angle 1$ and $\angle 2$ b. $\angle SRQ$ and $\angle TPQ$
c. $\angle SRV$ and $\angle VRQ$ d. $\angle STP$ and $\angle SRQ$

Use the figure below for Questions 12–15.



12. _____ is an acute angle.
- a. $\angle CAE$ b. $\angle DAE$
c. $\angle BAE$ d. $\angle BAD$
13. _____ is an obtuse angle.
- a. $\angle CAE$ b. $\angle DAE$
c. $\angle BAE$ d. $\angle BAD$
14. _____ is a right angle.
- a. $\angle CAE$ b. $\angle DAE$
c. $\angle BAE$ d. $\angle BAD$
15. _____ is a straight angle.
- a. $\angle CAE$ b. $\angle DAE$
c. $\angle BAE$ d. $\angle BAD$

Use this figure for Questions 16 and 17.



16. If $PQ = 3x + 7$, $QR = 4x - 2$, and $PR = 61$, the value of x is _____.
- a. 8 b. 31
c. 30 d. 7
17. If $PQ = 5x + 6$, $QR = 7x - 4$, and $\overline{PQ} \cong \overline{QR}$, then $PR =$ _____.
- a. 62 b. 5
c. 31 d. 7
18. $\overline{AB}$ has endpoints $A(7, -8)$ and $B(-3, 10)$. The coordinates of the midpoint of $\overline{AB}$ are _____.
- a. $(5, -9)$ b. $(4, 2)$
c. $(2, 1)$ d. $(10, -18)$
19. The midpoint of $\overline{CD}$ is $(7, -2)$. One endpoint is $C(9, -8)$. The other endpoint is _____.
- a. $D(11, -14)$ b. $D(-2, 6)$
c. $D(5, 4)$ d. $D(8, -5)$
20. The distance between points $G(-2, 3)$ and $H(4, -5)$ is _____.
- a. $2\sqrt{2}$ b. 100
c. $\sqrt{10}$ d. 10